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Velocity and Scour Prediction in River Bends

by *Colin R. Thorne*
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Prepared for Headquarters, U.S. Army Corps of Engineers

Velocity and Scour Prediction in River Bends

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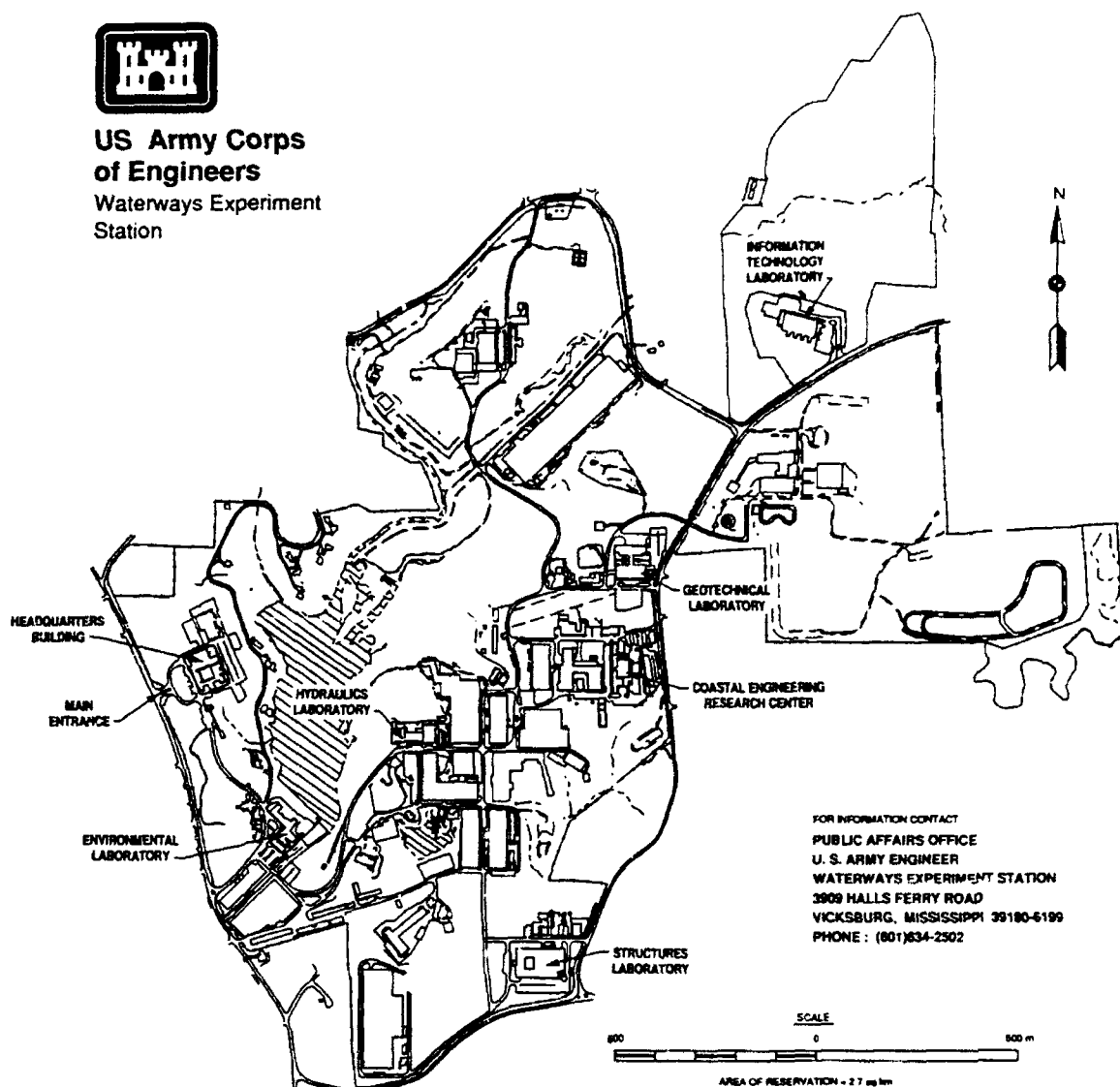
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PREFACE

The two studies reported herein were performed at Colorado State University, Fort Collins, CO, and the University of Nottingham, Nottingham, England, under contract to the US Army Engineer Waterways Experiment Station (WES) during the period October 1989 to June 1992. This investigation was sponsored by the Headquarters, US Army Corps of Engineers (HQUSACE), under the Flood Control Structures Research Program as part of Civil Works Investigation Work Unit No. 32544, "Riprap Toe and End Section Design," under HQUSACE Program Monitor, Mr. Tom Munsey.

This investigation was accomplished under the direction of Messrs. F. A. Herrmann, Jr., Director of the Hydraulics Laboratory (HL), WES; R. A. Sager, Assistant Director of HL; and G. A. Pickering, Chief of the Hydraulic Structures Division, HL. The Contracting Officer's Representative was Dr. S. T. Maynard, who was under the direct supervision of Mr. N. R. Oswalt, Chief of the Spillways and Channels Branch, Hydraulic Structures Division, HL.

At the time of publication of this report, Director of WES was Dr. Robert W. Whalin. Commander was COL Leonard G. Hassell, EN.

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ESTIMATION OF VELOCITY AND SHEAR STRESS
AT THE OUTER BANK IN RIVER BENDS

prepared for

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SUMMARY

Bank erosion is a serious problem to river engineers concerned with channel stabilization and navigation. Severe erosion often occurs at the outer bank in channel bends, where flow velocities adjacent to the bank are elevated due to the effects of curvature on channel flow. Eroding banks may be stabilized and protected from erosion using riprap. When selecting the appropriate size of stone to be used to protect a bank in a given bend, it is necessary to be able to predict the intensity of flow attack on the bank. This may be represented by either the near bank velocity or the boundary shear stress on the bank. This report deals with the development of improved methods to predict outer bank velocities and shear stresses. Two approaches are examined. The first uses a statistical treatment of observed data from natural and artificial channels to formulate predictive equations for the ratio of depth averaged longstream velocity over the toe of the outer bank and for the shear stress in that location. The second tests two analytical models of bend flow to gauge their accuracy and set limits to their applicability in predicting outer bank velocity.

The results show that several factors appear to influence outer bank velocity at a natural bend. Multivariate equations involving radius of curvature to width ratio, relative bend length, width to depth ratio, relative depth and bank angle are proposed to predict the ratio of outer bank toe velocity to average velocity. Simplified equations using only the *radius of curvature to width ratio* are also proposed. The configuration of the channel upstream of the bend is shown to be important, and separate approaches are formulated for bends downstream of straight and meandering reaches. For artificial channels Rc/w dominates the analysis, but it is also shown that the mobility of the bed strongly influences the outer bank velocity and shear stress.

Model tests reveal that the model developed by Bridge (1982) consistently predicts the observed outer bank toe velocity to within $\pm 15\%$. Errors grow alarmingly for bends with Rc/w values less than 2 and the model crashes for bends with $Rc/w < 1$. Odgaard's (1989) model tended to under predict outer bank velocity by between 5 and 40%. This was the case because the model did not predict outer bank scouring in bends with bed material coarser than medium sand. However, its application was limited because it predicted negative depths at the inner bank and crashed for long bends. In contrast to Bridge's model, Odgaard's model remained stable at very low Rc/w bends, errors remaining in the 5 to 40% range.

It is recommended that the results of this study be further tested and verified. However, on the basis of the results to date, the model developed by Bridge is recommended for use in bends with Rc/w values greater than 2. For very tight bends, Odgaard's model shows strong potential, but it must be modified to allow greater mobility and scour of coarse bed materials.

PREFACE

This project was sponsored by the Hydraulics Laboratory at the US Army Engineer Waterways Experiment Station (WES). The project was monitored by Dr. Steve Maynard. Dr. Maynard also made available field and laboratory data which were very useful in carrying out the work. Helpful advice was given by Mr. Randy Oswalt at WES on a number of occasions. Data assimilation and reduction were undertaken by a research assistant, Sue Reed, with great diligence and skill. The menu-driven programming of the Bridge and Odgaard bend flow models was performed by Andy Markham in the course of his graduate studies. The Principal Investigators wish to record their thanks to each of these individuals for their valuable contributions to the project.

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MAIN TEXT

Introduction

Serious bank erosion often occurs at the outer bank in meander bends. This erosion is driven by the natural tendency of river meanders to increase in amplitude and migrate downstream through time. The severity of flow attack on the bank is known to be controlled by the hydraulics of flow adjacent to the bank and especially the propensity for scour in the area of the bank toe. Conversely, the mechanics of failure and the sequence of events involved in the erosion, collapse and basal clean-out phases of bank retreat are closely related to the engineering properties of the bank materials and the bank stratigraphy. But the overall rate of retreat of the bank is known to be determined by the capacity of the near-bank flow to entrain and remove slumped bank materials, while continuing to erode the bank and trigger further failures (Thorne, 1982; Lapointe and Carson, 1986).

The importance of bank attack and toe scour by the flow have long been recognised, and their intensity has been found to be a function of the boundary shear stress acting on the bed and bank at the outer bank in a meander. But in practical terms the boundary shear stress is a particularly difficult parameter to predict accurately. Indeed, none specialists even find it difficult to visualize boundary shear stress. Consequently, it is desirable to relate the severity of bank attack and toe scour to less obscure flow descriptor, such as near-bank velocity. Some modelers even prefer to relate bank attack and retreat rates to near bank velocities instead of bank shear stress (Odgaard, 1990). Theory shows that near-bank velocity and boundary shear stress are in an, case closely related, although the relation between them is neither simple, or easily quantified for real world situations.

The preferred treatment to stabilize and protect the outer bank in a meander bend uses a blanket of loose stone called riprap. When using riprap it is necessary to select the appropriate size for the stone on the basis of the intensity of flow attack as represented by either the boundary shear stress on the outer bank or the flow velocity over the toe of the outer bank. Presently, this achieved using semi-empirical diagrams (Figs. 1 and 2).

The first (Fig. 1) predicts the ratio of velocity over the outer bank toe to average velocity in the approach channel (V_{toe}/V_{avg}) as a function of the radius of curvature to width ratio for the bend (R_c/w). The second (Fig. 2) predicts the ratio of outer bank shear stress to average boundary shear stress in the approach channel (t_b/t_o) as a function of the radius of curvature to width ratio for the bend.

Fig. 1 WES design diagram for prediction of outer bank velocity at a bend

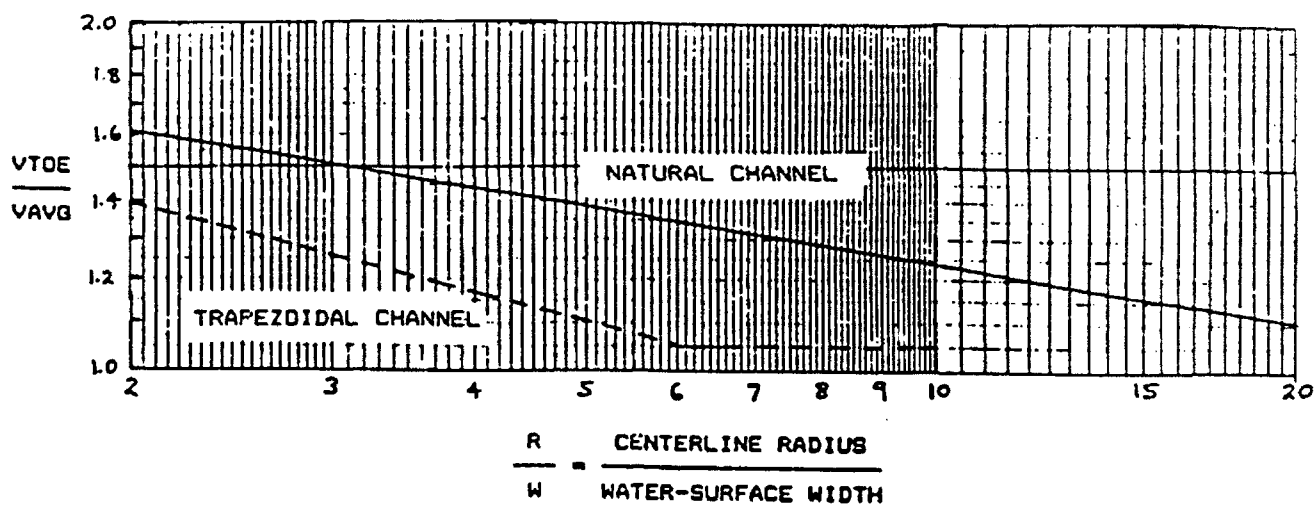
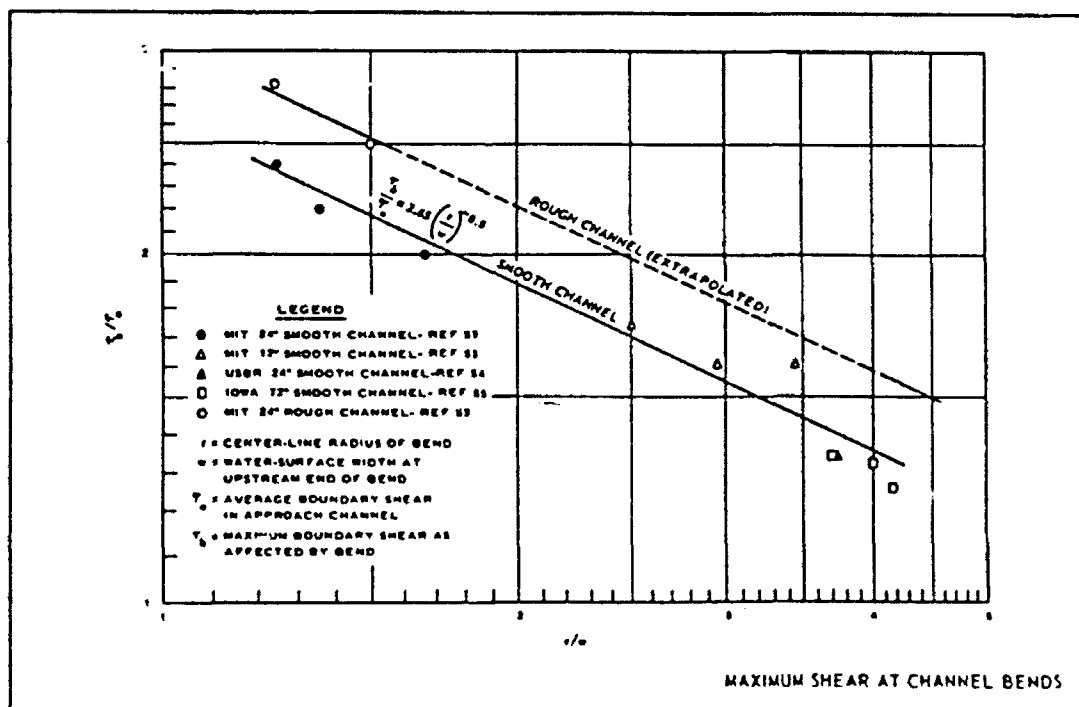


Fig. 2 WES design diagram for prediction of outer bank shear at a bend



The velocity diagram uses a logarithmic scale for the independent variable (Rc/w) and a linear scale for the dependent variable (V_{toe}/V_{avg}). Two lines are plotted, corresponding to natural channels (with asymmetrical cross-sections) and trapezoidal channels (with symmetrical cross-sections), respectively. The ratio of outer bank to mean velocity is markedly higher in natural than trapezoidal channels. Plotted as straight lines on a semi-log graph, these lines indicate logarithmic relations between (Rc/w) and (V_{toe}/V_{avg}) for the two types of channel. The equations of the lines are not given, but analysis of the graph suggests that they approximate to:

Natural Channels

$$\frac{V_{TOE}}{V_{AVG}} = 1.75 - 0.5 \log \left(\frac{R}{w} \right) \quad (1)$$

Trapezoidal Channels

$$\frac{V_{TOE}}{V_{AVG}} = 1.6 - 0.71 \log \left(\frac{R}{w} \right) \quad (2)$$

The shear stress diagram uses logarithmic axes for both independent (Rc/w) and dependent (t_b/t_o) variables. Again, two lines are plotted, this time corresponding to smooth and rough channels. All data appear to come from laboratory flumes, no data from natural rivers are included. Rough channels are found to have significantly higher stress ratios than smooth channels, for the same value of (Rc/w), although the line for rough channels is fitted to only two points and is heavily extrapolated. Plotted as straight lines on log-log graph, these lines indicate power function relations between (Rc/w) and (t_b/t_o). The equation for the smooth channel line is given on the diagram as:-

$$\frac{t_b}{t_o} = 2.65 \left(\frac{R}{w} \right)^{-0.5} \quad (3)$$

No equation for the rough channel line is given, but examination of the graph suggests that the line may be described by:-

$$\frac{t_b}{t_o} = 3.11 \left(\frac{R}{w} \right)^{-0.5} \quad (4)$$

While either diagram can give reasonable results when used with sound engineering judgement and with careful consideration of the limits

to its applicability, it is nonetheless desirable to develop improved procedures that better account for the parameters of flow hydraulics, boundary roughness and channel geometry that are believed to influence flow intensity at the outer bank in a meander bend. Several other aspects of bend geometry, channel shape and boundary roughness have been shown to influence bend flow patterns significantly on both theoretical and practical grounds (Thorne, 1978; Hooke and Harvey, 1983; Rais, 1984; Lapointe and Carson, 1986; Pizzuto, 1987; Thorne and Osman, 1988; Odgaard, 1989), and a method which uses only a single parameter to characterize the bend, ignoring all others, cannot account for these effects.

Objectives

The objectives of this study are to develop improved analytical techniques to estimate the velocity and shear stress distributions at the outer bank in a river bend. The approach adopted is to examine these distributions as functions of the planform and cross-sectional geometry of the bend, the nature of the bed and bank materials, and the planform and average flow parameters in the approach channel.

The primary objective is to concentrate on defining maximum values of depth averaged velocity that occur in the bend along the outer bank (that is over the toe of revetted banks). The second aim is to produce the equivalent relationships for boundary shear stress at the outer bank in a meander bend.

Emphasis is placed on basing the relationships on parameters readily available to design engineers, rather than variables such as "centerline mean velocity" which although theoretically significant, are usually unknown and which would themselves be difficult to predict or estimate.

Approaches Adopted

Broadly, two approaches have been used. The first is based on statistical analysis of a data base on bend flow assembled from published and unpublished reports of studies made on rivers and in laboratory flumes all over the world. The second attempts a more theoretical approach, being based on application of three recently developed mathematical models of bend flow hydraulics. There are advantages and disadvantages to both approaches and these are discussed in the sections concerned with the Final Discussion and Conclusions.

Data-Based Approach

Sources of Data

Data were obtained from a number of diverse sources. The sources actually used are listed in Appendix A. The initial data came from studies undertaken by the Principal Investigators and their colleagues at Colorado State University, London University, UK and the University of East Anglia, UK. These data were readily to hand and included all of the parameters necessary for this analysis. They required only a little time and effort to assemble.

The second source of data was from researchers known to be working on bend flow problems and with whom the Principal Investigators have good working relationships. In response to requests from the PI's or their research associates, copies of research reports and published articles containing full data sets were supplied by these individuals, mostly in a timely fashion. This allowed easy extraction of the relevant parameters. In cases where a particular measurement was not reported, telephone calls to the original researchers usually elicited the missing information.

The third source of data was from papers published in professional and learned journals. This proved to be the least satisfactory source. Journal papers almost never contain full data sets, and published summary diagrams of the distribution of parameters such as depth-averaged velocity are too small to be used for data extraction with any degree of accuracy or precision. The addresses given in articles are often incomplete or out of date and telephone and FAX numbers are omitted. Most authors were extremely slow to respond to written enquiries sent by ordinary mail and some seemed reluctant to part with data at all. These problems led to several promising leads being reluctantly abandoned and data sets excluded from the analysis.

The data set which has resulted is then not universal in its scope. It does, however, contain only data which the Principal Investigators opinions is sound and complete. The range of sizes and types of channel encompassed is large and there is a sufficient number of entirely independent data sets to support the statistical analysis. Consequently, it is probable that the addition of a few further data is unlikely to materially alter the overall distribution of data or the outcome of the analyses.

Data base

The basic data assembled in this study are listed in Tables 1, 2 and 3, for Natural Rivers, Trapezoidal Channels and Rectangular Channels respectively. The published and unpublished sources of data are listed separately in the reference section of this report.

TABLE 1 - BASIC DATA FOR NATURAL RIVERS

RESEARCHER	RIVER	SITE	BEND NUMBER	RADIUS OF CURVATURE (m)	BEND LENGTH (m)	WIDTH (m)	MEAN DEPTH (m)	X-SECT SHAPE	OUTER BANK ANGLE (Degrees)	OUTER BANK ROUGHNESS	DSO (m)	BEDFORMS	APPROACH CHANNEL	AVERAGE VELOCITY (m/s)	DEPTH-AVE TOE VELOCITY (m/s)
Mathison/Thorne	Fall	Ranch B	1	23.50	41.00	8.20	0.63	N	60	R	0.0140	P	S	0.33	0.80
Thorne et al.	Fall	Ranch A	1	10.30	19.00	12.50	0.74	N	75	R	0.0097	P	M	0.57	0.80
Thorne et al.	Fall	Ranch A	2	8.30	14.00	11.00	0.92	N	85	R	0.0098	P	M	0.46	0.60
Thorne et al.	Fall	Ranch A	3	8.71	21.00	9.90	0.94	N	70	R	0.0130	P	S	0.71	1.10
Mathison/Thorne	Rolling	Longhorn	1	21.00	64.00	12.00	1.30	N	60	R	0.0130	P	S	1.13	1.35
C.R. Thorne	Fall	Ranch 1	1	11.00	50.80	8.80	0.89	N	65	I	0.0010	R,D	S	0.31	0.80
C.R. Thorne	Fall	Ranch 1	2	13.50	48.00	10.60	0.66	N	64	I	0.0010	R,D	S	0.58	0.74
D. Anthony	Fall	Ranch 4	1	13.75	60.40	10.81	0.79	N	63	I	0.0042	D	S	0.48	0.70
J.S. Bridge	South Fork	Chen Cove	1	67.10	115.00	10.81	1.22	N	58	I	0.0011	R,D	S	0.48	0.69
N.O. Bhowmik	Kakabadi	Ranch 1	1	301.80	552.25	38.10	3.77	N	34	R	0.0022	D	S	0.84	1.05
N.O. Bhowmik	Kakabadi	Ranch 1	2	291.70	246.60	45.40	3.59	N	45	R	0.0005	R,D	M	0.86	0.95
N.O. Bhowmik	Kakabadi	Ranch 1	3	136.60	204.20	34.30	4.01	N	30	R	0.0009	D	M	0.84	1.03
N.O. Bhowmik	Kakabadi	Ranch 1	4	40.80	103.73	34.30	3.84	N	46	R	0.0054	D	M	0.62	0.80
N.O. Bhowmik	Kakabadi	Ranch 1	5	32.00	85.45	39.90	3.48	N	58	R	0.0057	D	S	0.69	0.93
N.O. Bhowmik	Kakabadi	Ranch 2	2	380.40	505.99	48.60	3.41	N	61	R	0.0052	D	S	0.61	0.83
N.O. Bhowmik	Kakabadi	Ranch 2	3	91.40	298.55	47.10	3.68	N	59	R	0.0023	D	M	0.61	0.74
N.O. Bhowmik	Kakabadi	Ranch 2	4	213.40	371.98	45.40	3.69	N	51	R	0.0048	D	S	0.61	0.70
N.O. Bhowmik	Kakabadi	Ranch 2	5	95.00	41.00	25.00	0.65	N	90	R	0.0017	P	S	0.94	0.98
Balhuur/Thorne	Screen	Mass Maw	1	44.00	51.50	9.10	0.81	N	90	R	0.0030	P	S	1.35	1.60
S. Maynard	Misouri	Rickety Bridge	1	3623.50	4023.00	202.50	5.55	N	77	I	0.0003	R,D	M	1.32	1.46
S. Maynard	Misouri	Brown Bend	2	3718.50	4345.00	199.50	5.50	N	22	I	0.0003	R,D	S	1.36	1.53
S. Maynard	Misouri	Snyder Bend	3	2000.25	4023.25	200.00	5.65	N	23	I	0.0003	R,D	S	1.27	1.62
S. Maynard	Misouri	Cheney Point Bend	4	1952.65	4827.90	209.00	5.30	N	28	I	0.0003	R,D	M	1.35	1.55
S. Maynard	Misouri	Windsor Bend	5	1857.38	1609.30	223.75	5.00	N	25	I	0.0003	R,D	M	1.42	1.60
S. Maynard	Misouri	Upper Omaha Mission	6	2714.63	4023.50	194.00	5.45	N	22	I	0.0003	R,D	M	1.40	1.81
S. Maynard	Misouri	Middle Omaha Mission	7	2047.88	2413.95	213.50	5.45	N	28	I	0.0003	R,D	M	1.40	1.55
S. Maynard	Misouri	Lower Omaha Mission	8	2143.13	1609.30	213.50	5.03	N	36	I	0.0003	R,D	M	1.37	1.55
S. Maynard	Misouri	Upper Missouri Bend	9	2619.38	5632.55	223.00	5.48	N	26	I	0.0003	R,D	M	1.47	1.67
S. Maynard	Misouri	Lower Missouri Bend	10	4524.38	5632.55	209.25	5.15	N	21	I	0.0003	R,D	S	1.45	1.54
S. Maynard	Misouri	Blackbird Bend	11	2381.25	4023.25	199.75	5.35	N	22	I	0.0003	R,D	M	1.42	1.69
S. Maynard	Misouri	Terrill Bend	1	16.00	47.60	5.88	0.50	N	74	I	0.0010	D	S	0.43	0.55
de Vriend/Gelder	Damned	The Netherlands	1	14.50	31.70	4.00	0.50	N	64	I	0.0010	D	S	0.43	0.61
Dieckhoff	Dead	The Netherlands	2	8.00	25.00	4.00	0.40	N	67	I	0.0007	R,D	S	0.55	0.75
A.J. Olgard	Muddy Creek	Wyoming	1	233.00	560.00	48.00	2.05	N	47	R	0.0005	R,D	S	1.25	1.60

R = Rough
I = Intermediate
S = Smooth

N = Normal

R = Rough
I = Intermediate
S = Smooth

R = Rough
I = Intermediate
S = Smooth

R = Rough
I = Intermediate
S = Smooth

R = Rough
I = Intermediate
S = Smooth

R = Rough
I = Intermediate
S = Smooth

R = Rough
I = Intermediate
S = Smooth

R = Rough
I = Intermediate
S = Smooth

R = Rough
I = Intermediate
S = Smooth

TABLE 2 - BASIC DATA FOR TRAPEZOIDAL CHANNELS

RESEARCHER	SITE	BEND NUMBER	RADIUS OF CURVATURE (m)	BEND LENGTH (m)	WIDTH (m)	MEAN DEPTH (m)	OUTERBANK ANGLE (Degrees)	OUTERBANK ROUGHNESS $\frac{S}{H/R}$	DSO (m)	BEDFORMS $\frac{R, D, P}{M}$	APPROACH CHANNEL $\frac{S, M, B}{M}$	AVERAGE VELOCITY (m/s)	DEPTH-AVE TOB VELOCITY (m/s)	SHEAR STRESS RATIO
A.J. Odgaard	Iowa Hyd. Res.	1	13.11	41.18	2.44	0.15	56	S	0.0003	R.D.	S	0.45	0.55	
A.J. Odgaard	WES	1	13.11	41.17	2.44	0.10	34	S	0.0003	R.D.	S	0.59	0.73	
WES	H.L.S.D.	1	15.24	23.93	6.76	0.78	27	I	0.0341	P	S	1.04	1.36	
WES	H.L.S.D.	2	15.24	35.91	6.70	0.77	27	I	0.0341	P	M	1.07	1.39	
WES	H.L.S.D.	3	15.24	23.92	6.72	0.77	27	I	0.0341	P	M	1.06	1.50	
WES	H.L.S.D.	1	8.05	14.06	2.69	0.14	27	I	0.0127	P	S	0.57	0.73	
WES	H.L.S.D.	2	8.05	14.06	2.69	0.14	27	I	0.0127	P	S	0.57	0.68	
D. Mueller	DOBR	1	4.87	1.27	1.30	0.23	34	S	0.0001	P	S	0.34		1.33
D. Mueller	Univ. of Iowa	1	8.53	13.39	2.29	0.23	45	S	0.0001	P	S	0.57		1.30
D. Mueller	MIT	1	1.52	1.60	0.91	0.08	27	S	0.0001	P	S	0.41		2.00
D. Mueller	MIT	2	1.54	1.61	1.23	0.15	27	S	0.0001	P	S	0.36		2.80
Ippen & Drinker	MIT	4	1.50	1.60	1.22	0.11	27	S	0.0001	P	S	0.58		2.00
Ippen & Drinker	MIT	7	1.78	1.86	0.71	0.07	27	S	0.0001	P	S	0.43		1.75
Ben-Chia Yen	Univ. Iowa	1	8.53	13.40	2.05	0.10	45	S	0.0001	P	S	0.82	0.89	1.00
Ben-Chia Yen	Univ. Iowa	2	8.53	13.40	2.15	0.15	45	S	0.0001	P	S	0.69	0.71	1.00
Ippen & Drinker	MIT	1	1.78	1.86	0.61	0.08	27	S	0.0001	P	S	0.36	0.42	1.60

NOTE: R = Rough I = Intermediate S = Smooth
 R = Rippled D = Dunes Smooth P = Plane
 M = Meandering B = Braided
 S = Straight

TABLE 3 - BASIC DATA FOR RECTANGULAR CHANNELS

RESEARCHER	SITE	BEND NUMBER	RADIUS OF CURVATURE (m)	BEND LENGTH (m)	WIDTH (m)	MEAN DEPTH (m)	OUTER BANK ANGLE (Degrees)	OUTER BANK ROUGHNESS ---S/R---	DSO (m)	BEDFORMS ---L/D/P---	APPROACH CHANNEL ---S/M/B---	AVERAGE VELOCITY (m/s)	DEPTH-AVE TOE VELOCITY (m/s)	SEAR STRESS RATIO
Choudhary & Narendran	Banaras, India	1	0.80	0.42	0.96	0.1920	90	S	0.0001	P	S			1.20
"	Banaras, India	2	0.80	0.84	0.96	0.19	90	S	0.0001	P	S			1.20
"	Banaras, India	3	0.80	0.42	0.96	0.10	90	S	0.0001	P	S			1.10
"	Banaras, India	4	0.80	0.84	0.96	0.10	90	S	0.0001	P	S			1.20
Vishwesh & Gupta	Rodates, India	1	1.80	1.89	0.60	0.27	90	S	0.0020	P	S	0.37		2.36
"	Rodates, India	2	1.80	1.89	0.60	0.07	90	S	0.0020	P	S	0.54		2.46
"	Rodates, India	3	1.80	1.89	0.60	0.21	90	S	0.0020	P	S	0.35		2.46
For & Ball	Leeds, UK	1	1.07	3.35	0.31	0.15	90	S	0.0001	P	S	0.33	0.37	
Rozovskii	USSR	1	0.80	2.51	0.80	0.06	90	S	0.0001	D	S	0.36	0.36	
Kikawa et al.	Japan	1	4.50	14.14	1.00	0.05	90	S	0.0009	D	S	0.40	0.32	
Kikawa et al.	Japan	2	4.50	14.14	1.00	0.06	90	S	0.0009	D	S	0.45	0.55	
Kikawa et al.	Japan	3	4.50	14.14	1.00	0.06	90	S	0.0009	D	S	0.48	0.60	
Struikema et al.	Deilt, Holland	1	12.00	29.32	1.50	0.08	90	S	0.0005	D	S	0.39	0.46	
Struikema et al.	Deilt, Holland	2	12.00	29.32	1.50	0.10	90	S	0.0005	D	S	0.41	0.48	
Hooke	Uppsala, Sweden	1	2.36	5.77	1.00	0.07	90	S	0.0003	P	S	0.28	0.37	2.00
Hooke	Uppsala, Sweden	2	2.36	5.77	1.00	0.10	90	S	0.0003	P	S	0.38	0.38	1.50
Hooke	Uppsala, Sweden	3	2.36	5.77	1.00	0.09	90	S	0.0003	P	S	0.38	0.38	1.50
Hooke	Uppsala, Sweden	4	2.36	5.77	1.00	0.13	90	S	0.0003	P	S	0.39	0.39	1.75
Bray & Ho	Fredrickson, Can	1	3.00	3.14	1.00	0.15	90	S	0.0001	P	S	0.55	0.55	1.60
Bray & Ho	Fredrickson, Can	5	3.00	3.14	0.67	0.10	90	S	0.0001	P	S	0.38	0.38	1.40
Bray & Ho	Fredrickson, Can	7	3.00	3.14	0.33	0.15	90	S	0.0001	P	S	0.38	0.38	1.60
Bray & Ho	Fredrickson, Can	9	3.00	3.14	0.33	0.05	90	S	0.0001	P	S	0.24	0.24	1.31
Onishi, Jain & Kennedy	IBR	1	8.53	13.41	2.34	0.13	90	S	0.0003	D	S	0.54	0.71	
"	IBR	2	9.12	14.32	1.17	0.13	90	S	0.0003	D	S	0.54	0.61	
McCrea & Bray	Fredrickson, Can	1	3.00	3.14	1.00	0.20	90	S	0.0001	P	S	0.30	0.35	
McCrea & Bray	Fredrickson, Can	2	3.00	3.14	1.00	0.20	50	S	0.0001	P	S	0.30	0.35	
Nash & Townsend	Calgary, Canada	1	0.90	0.71	0.30	0.04	90	S	0.0007	P	S	0.30	0.35	1.00
"	Calgary, Canada	2	0.90	0.94	0.30	0.04	90	S	0.0007	P	S	0.30	0.35	2.40
de Vriend & Koch	LFM	1	4.25	7.85	1.70	0.17	90	R	0.0001	P	S	0.66	0.81	
de Vriend & Koch	LFM	2	4.25	7.85	1.70	0.17	90	R	0.0001	P	S	0.60	0.75	
de Vriend & Koch	Delft Hydraulic Lab.	1	50.00	72.00	6.00	0.25	90	S	0.0001	P	S	0.41	0.45	
de Vriend & Koch	Delft Hydraulic Lab.	2	50.00	72.00	6.00	0.25	90	S	0.0001	P	S	0.40	0.47	
C. L. Yen	IBR	1	8.53	13.40	2.34	0.12	90	S	0.0003	P	S	0.32	0.40	
Hicks, Jin, & Stoffer	Alberta University	A1	3.66	17.2	1.07	0.08	18	S	0.0001	P	S	0.44	0.56	
Hicks, Jin, & Stoffer	Alberta University	B1	3.66	17.2	1.07	0.09	27	S	0.0001	P	S	0.46	0.55	

Explanation

R = Rough
 I = Intermediate
 S = Smooth
 K = Ripples
 D = Dunes
 M = Meandering
 B = Braided
 S = Straight
 M = Meandering
 B = Braided

The basic data were used to derive parameters of bend geometry and hydraulic roughness which could affect the pattern of flow through the bend. The derived data are listed in Tables 4, 5 and 6 for Natural Rivers, Trapezoidal Channels and Rectangular Channels respectively.

An important aspect of any experimentally based study is to identify the range of each variable observed. When applying relationships based on the experimental results, these ranges must set the limits to the applicability of the relations. It is highly speculative and very risky to apply any empirical relationship outside the range of data from which it has been developed and tested. The range of each of the variables is listed in Data Tables 7, 8 and 9 for Natural, Trapezoidal and Rectangular Channels, respectively.

TABLE 4 - DERIVED DATA FOR NATURAL RIVERS

RESEARCHER	RIVER	SITE	BEND NUMBER	R/W	L/W	W/H	4D50	OUTERBANK ANGLE α (angle)	OUTERBANK ROUGHNESS $\dots S/\Delta \dots$	BEDFORMS $\dots R/D/S/P \dots$	APPROACH CHANNEL $\dots S/M/B \dots$	View/Bar
Madhuen/Thorne	Fall	Reach B	1	2.87	5.00	12.62	46.4	0.867	R	P	S	1.51
Thorne et al.	Fall	Reach A	1	0.82	1.52	16.45	78.4	0.967	R	P	M	1.40
Thorne et al.	Fall	Reach A	2	0.75	1.45	11.96	242.1	0.996	R	P	M	1.30
Thorne et al.	Fall	Reach A	3	0.88	2.12	10.31	73.8	0.94	R	P	S	1.35
Madhuen/Thorne	Rodrig	Loughran	1	0.88	5.50	9.23	100.0	0.87	R	P	S	1.19
C.R. Thorne	Fall	Reach 1	1	1.25	5.77	9.89	990.0	0.99	1	R/D	M	1.57
C.R. Thorne	Fall	Reach 1	2	1.27	4.53	16.06	660.0	0.999	1	R/D	M	1.28
D. Anthony	Fall	Reach 4	1	1.27	5.59	13.68	190.4	0.891	1	D	S	1.46
J.S. Bridge	South Esk	Clon Cove	1	2.92	5.00	18.85	677.8	0.848	1	R/D	S	1.44
N.G. Shownitz	Kaskadi	Reach 1	1	7.92	8.72	10.11	1713.6	0.559	R	D	S	1.25
N.G. Shownitz	Kaskadi	Reach 1	2	6.58	6.31	12.65	7180.0	0.707	R	R/D	M	1.10
N.G. Shownitz	Kaskadi	Reach 1	3	3.76	5.63	9.05	4455.6	0.5	R	D	M	1.23
N.G. Shownitz	Kaskadi	Reach 1	4	1.12	2.86	9.45	1129.4	0.719	R	D	M	1.29
N.G. Shownitz	Kaskadi	Reach 1	5	0.80	2.14	11.47	610.5	0.848	R	D	S	1.35
N.G. Shownitz	Kaskadi	Reach 2	2	7.83	10.41	14.25	655.8	0.875	R	D	S	1.36
N.G. Shownitz	Kaskadi	Reach 2	3	1.94	6.34	12.80	1472.0	0.799	R	D	M	1.21
N.G. Shownitz	Kaskadi	Reach 2	4	4.70	8.19	12.30	768.8	0.777	R	D	S	1.15
Severn	Severn	Moss Mill	1	3.80	1.64	38.46	20.5	0.985	R	P	S	1.04
Bethune/Thorne	Severn	Ridley Bridge	1	4.84	3.57	10.46	13.8	1	R	P	S	1.11
S. Maynard	Miscouri	Brown Bend	1	17.90	19.87	36.49	18500.0	0.454	1	R/D	M	1.11
S. Maynard	Miscouri	Snyder Bend	2	16.23	21.78	36.27	18333.3	0.375	1	R/D	S	1.13
S. Maynard	Miscouri	Glover Point Bend	3	10.00	20.12	35.40	18833.3	0.391	1	R/D	S	1.28
S. Maynard	Miscouri	Winebago Bend	4	9.34	23.10	39.43	17666.7	0.469	1	R/D	M	1.15
S. Maynard	Miscouri	Upper Omaha Mission	5	8.23	7.13	45.15	16666.7	0.423	1	R/D	M	1.19
S. Maynard	Miscouri	Middle Omaha Mission	6	13.85	20.53	35.96	18166.7	0.375	1	R/D	M	1.26
S. Maynard	Miscouri	Lower Omaha Mission	7	9.64	11.36	38.99	18166.7	0.469	1	R/D	M	1.11
S. Maynard	Miscouri	Upper Monona Bend	8	9.26	6.95	46.02	16766.7	0.588	1	R/D	M	1.13
S. Maynard	Miscouri	Lower Monona Bend	9	11.75	25.26	40.69	18266.7	0.438	1	R/D	M	1.14
S. Maynard	Miscouri	Blackbird Bend	10	21.62	26.92	40.63	17166.7	0.558	1	R/D	S	1.06
S. Maynard	Miscouri	Therle Bend	11	11.92	20.14	38.05	17500.0	0.375	1	R/D	M	1.19
de Vriend/Caldorf	Donned	The Netherlands	1	2.72	8.10	11.76	500.0	0.961	1	D	S	1.28
de Vriend/Caldorf	Donned	The Netherlands	2	2.42	5.28	12.00	500.0	0.999	1	D	S	1.45
Dietrich & Smith	Muddy Creek	Wyoming	1	2.00	6.25	10.00	571.4	0.92	S	R/D	M	1.36
A.J. Odgaard	E. Nishabone	Iowa	1	4.85	11.67	23.41	4100.0	0.731	R	R/D	S	1.28

TABLE 5 - DERIVED DATA FOR TRAPEZOIDAL CHANNELS

RESEARCHER	CHANNEL	SITE	BEND NUMBER	Re/W	L/W	w/d	d/D50	V _{ave} /V _{bar}	SHEAR STRESS RATIO
A.J. Odgaard	Lab. Channel	Inst Hyd Res.	1	5.37	16.88	16.27	500.0	1.22	
A.J. Odgaard	Lab. Channel	WES	1	5.37	16.87	24.40	333.3	1.24	
WES	RFT (I)	H.L.S.D.	1	2.25	3.54	8.67	20.5	1.31	
WES	RFT (II)	H.L.S.D.	2	2.27	3.56	8.70	20.2	1.30	
WES	RFT (III)	H.L.S.D.	3	2.27	3.56	8.73	20.2	1.42	
WES	RFT (IV)	H.L.S.D.	1	2.99	5.23	19.21	11.0	1.28	
WES	RFT (V)	H.L.S.D.	2	2.99	5.23	19.21	11.0	1.19	
D. Mueller	Lab. Channel	USSR	1	3.75	0.91	5.70	2280.0		1.33
D. Mueller	Lab. Channel	Univ. of Iowa	1	3.72	5.85	10.00	2290.0		1.30
D. Mueller	Lab. Channel	MIT	1	1.67	1.76	11.38	800.0		2.00
D. Mueller	Lab. Channel	MIT	2	1.25	1.31	8.20	1500.0		2.80
Ippen & Drinker	Lab. Channel	MIT	4	1.23	1.31	10.70	1140.0		2.00
Ippen & Drinker	Lab. Channel	MIT	7	2.51	2.62	10.14	700.0		1.75
Ben-Chie Yen	Lab. Channel	Univ. Iowa	1	4.16	6.54	20.10	1020.0	1.09	1.00
Ben-Chie Yen	Lab. Channel	Univ. Iowa	2	3.97	6.23	14.83	1450.0	1.05	1.00
Ippen & Drinker	Lab. Channel	MIT	1	2.91	3.04	7.95	770.0	1.17	1.60

TABLE 6 - DERIVED DATA FOR RECTANGULAR CHANNELS

RESEARCHER	CHANNEL	SITE	BEND NUMBER	Re/w	L/w	w/d	d/D50	BEDFORMS --R.D.P.--	V _{ave} /V _{bar}	SHEAR STRESS RATIO
Choudhary & Narmasthan	Lab. Channel	Bombay, India	1	0.83	0.4365	3.0000	1920.0	P		1.20
	Lab. Channel	Bombay, India	2	0.83	0.87	5.00	1920.0	P		1.20
	Lab. Channel	Bombay, India	3	0.83	0.44	10.00	960.0	P		1.10
	Lab. Channel	Bombay, India	4	0.83	0.87	10.00	960.0	P		1.20
Vannoy & Geste	U.P. Irrigation Research Institute	Roorkee, India	1	3.00	3.15	2.26	132.5	P		2.36
		Roorkee, India	2	3.00	3.15	9.16	32.8	P		2.46
		Roorkee, India	3	3.00	3.15	2.85	105.2	P		2.46
For & Ball	Lab. Channel	London, UK	1	3.51	10.98	2.00	1524.0	P	1.12	
Rozovski	IIHR	USSR	1	1.00	3.14	13.33	600.0	D		1.38
Kikawa et al.	Lab. Channel	Japan	1	4.50	14.14	20.00	35.6	D	1.29	
Kikawa et al.	Lab. Channel	Japan	2	4.50	14.14	18.18	61.1	D	1.22	
Kikawa et al.	Lab. Channel	Japan	3	4.50	14.14	15.87	70.0	D	1.25	
Struikama et al.	Lab. Channel	Delft, Holland	1	8.00	19.55	18.75	177.8	D	1.14	
Struikama et al.	Lab. Channel	Delft, Holland	2	8.00	19.55	15.00	222.2	D	1.17	
Hooke	Lab. Channel	Uppsala, Sweden	1	2.36	5.77	13.70	243.3	P		2.00
Hooke	Lab. Channel	Uppsala, Sweden	2	2.36	5.77	10.53	316.7	P		1.50
Hooke	Lab. Channel	Uppsala, Sweden	3	2.36	5.77	10.87	306.7	P		1.50
Hooke	Lab. Channel	Uppsala, Sweden	4	2.36	5.77	7.81	426.7	P		1.75
Bry & Ho	Lab. Channel	Frederickton, Can	1	3.00	3.14	6.67	1500.0	P		1.60
Bry & Ho	Lab. Channel	Frederickton, Can	5	4.50	4.71	6.67	1000.0	P		1.40
Bry & Ho	Lab. Channel	Frederickton, Can	7	9.01	9.43	2.22	1500.0	P		1.60
Bry & Ho	Lab. Channel	Frederickton, Can	9	9.01	9.43	6.66	500.0	P		1.31
Onishi, Jain & Kennedy	Lab. Channel	IIHR	1	3.65	5.73	18.00	520.0	D	1.31	
	Lab. Channel	IIHR	2	7.79	12.24	9.00	520.0	D	1.13	
McCrea & Bry	Lab. Channel	New Brunswick	1	3.00	3.14	5.00	2000.0	P	1.17	
McCrea & Bry	Lab. Channel	New Brunswick	2	3.00	3.14	5.00	2000.0	P	1.17	
Nouh & Townsend	Lab. Channel	Calgary, Can	1	3.00	2.37	7.50	57.1	P		1.80
	Lab. Channel	Calgary, Can	2	3.00	3.13	7.50	57.1	P		2.40
de Vriend & Koch	Lab. Channel	LFM	1	2.50	4.62	10.00	1700.0	P	1.23	
de Vriend & Koch	Lab. Channel	LFM	2	2.50	4.62	10.00	4.3	P	1.25	
de Vriend & Koch	Lab. Channel	Delft Hydraulic Lab.	1	8.33	12.00	24.00	2500.0	P	1.10	
de Vriend & Koch	Lab. Channel	Delft Hydraulic Lab.	2	8.33	12.00	24.00	2500.0	P	1.18	
C. L. Yen	Lab. Channel	IIHR	1	3.65	5.73	20.03	417.1	P	1.25	1.20
Hicks, Jin, & Stedler	Lab. Channel	Alberta University	A1	3.42	16.07	13.38	800.0	P	1.27	
Hicks, Jin, & Stedler	Lab. Channel	Alberta University	B1	3.42	16.07	12.30	870.0	P	1.20	

Table 7 - Range of Variables for Natural Channels

Measured Variables		
Variable	Units	Range
Radius of Curvature	meters	8 - 4,525
Bend Length	meters	16 - 5,633
Width	meters	4 - 232
Average Depth	meters	0.4 - 5.65
Outer Bank Angle	degrees	21 - 90
Outer Bank Roughness	- -	Rough-Intermediate
Median Bed Material Size	millimeters	0.3 - 63
Bedforms	- -	Plane - Dunes
Approach Channel	- -	Straight-Meandering
Average Velocity	meters/second	0.42 - 1.47
Depth-averaged Toe Velocity	meters/second	0.55 - 1.81
Derived Variables		
R/w	- -	0.75 - 21.6
L/w	- -	1.45 - 26.9
w/d	- -	9.05 - 46.1
d/D50	- -	13.8 - 18,833
Vtoe/Vavg	- -	1.04 - 1.57

Table 8 - Range of Variables for Trapezoidal Channels

Measured Variables		
Variable	Units	Range
Radius of Curvature	meters	1.5 - 15.24
Bend Length	meters	1.27 - 41.18
Width	meters	0.61 - 6.76
Average Depth	meters	0.07 - 0.78
Outer Bank Angle	degrees	27 - 56
Outer Bank Roughness	- -	Smooth-Intermediate
Median Bed Material Size	millimeters	Smooth - 38.1
Bedforms	- -	Plane-Dunes
Approach Channel	- -	Straight-Meandering
Average Velocity	meters/second	0.34 - 1.07
Depth-averaged Toe Velocity	meters/second	0.42 - 1.50

Derived Variables

R/w	--	1.23 - 4.16
L/w	--	1.31 - 16.88
w/d	--	5.70 - 24.40
d/D50	--	11.0 - 2290
V _{toe} /V _{avg}	--	1.03 - 1.42
T _{toe} /T _{avg}	--	1.00 - 2.80

Table 9 - Range of Variables for Rectangular Channels

Measured Variables

Variable	Units	Range
Radius of Curvature	meters	0.8 - 50
Bend Length	meters	0.42 - 72.0
Width	meters	0.30 - 6.00
Average Depth	meters	0.05 - 0.27
Outer Bank Angle	degrees	18 - 90
Outer Bank Roughness	--	Rough-Smooth
Median Bed Material Size	millimeters	Smooth - 40
Bedforms	--	Plane-Dunes
Approach Channel	--	Straight
Average Velocity	meters/second	0.24 - 0.66
Depth-averaged Toe Velocity	meters/second	0.35 - 0.81

Derived Variables

R/w	--	0.83 - 9.01
L/w	--	0.44 - 19.55
w/d	--	2.22 - 24.0
d/D50	--	4.3 - 2,500
V _{toe} /V _{avg}	--	1.10 - 1.38
T _{toe} /T _{avg}	--	1.20 - 2.46

Examination of Data

Before undertaking any advanced analysis or statistical treatment of data, it is important to examine the data carefully in the light of existing knowledge and theory. This allows the researcher to identify expected and unexpected trends and relationships, and establishes the analytical framework for the formal treatment of the data. This, fairly lengthy, procedure is essential if the resulting relationships are to have physical as well as statistical significance.

The first step was to establish how the data collected in this study plotted in relation to the design curve developed by the US Army Engineer Waterways Experiment Station. Hence, a semi-logarithmic plot of (Rc/w) versus (V_{toe}/V_{avg}) was produced for the Natural River data, with the WES design curve marked on (Fig. 3a). The design line does not pass through the points, but does form a good upper bound to the data with the exception of only three out of 34 points. Thus, it may be concluded that the WES design curve represents a reasonable, but rather conservative approach to the estimation of (V_{toe}/V_{avg}) in natural channels. This is essential so that in the final design, the size of riprap specified is always on the safe side. A regression line through the scatter of the points for V_{toe}/V_{avg} could be used, but this would require that a factor of safety be introduced in the relationship between the critical local velocity for entrainment and the size of stone used in a revetment. Present WES preference is to position the design line as an upper bound to the data, so that all of the zone of uncertainty is on one side of the line (Oswald, personal communication, March 1990).

However, there is considerable scatter in the data, and this deserves comment. Partly, it is a result of the methods used to collect the data. Usually, velocities were measured at a finite number of cross-sections around each bend. In some studies many sections were used (up to seven per bend), but in others only a few (less than three) were used. Outer bank velocities at intermediate points between sections were not measured. Consequently, there is no guarantee that the actual maximum outer bank in a bend would be observed in any study. Indeed, in studies with only a few sections, it is highly probable that the outer bank maximum velocity for a bend would not be measured. It is therefore to be expected that field data should plot either close to or below a line defining the maximum possible ratio of outer bank to average velocity. However, even for bends with multiple measured sections, the data often plot well below the WES line. This suggests that there may be further variables affecting the velocity ratio which are unaccounted for in the WES analysis.

Points for bends of very low Rc/w values reveal that the monotonic increase in V_{toe}/V_{avg} observed as Rc/w decreases may cease at an Rc/w of about 2. For Rc/w values less than 2, the data show a wide range of V_{toe}/V_{avg} values, but the velocity ratio never exceeds 1.6. This accords with other recent studies of bend flow in very tightly curved bends, which has shown that both outer bank scour pool depth and outer bank retreat rate may actually decrease with decreasing Rc/w for bends with Rc/w less than 2 (Biedenharn et al., 1989; Thorne, 1989). This is not unexpected theoretically, as there is a major discontinuity in the way the pattern of bend flow responds to increasing bend tightness at Rc/w of between 2 and 3 (Bagnold, 1960). Further data and analyses are required to confirm this tentative finding.

Fig. 3a Natural Rivers

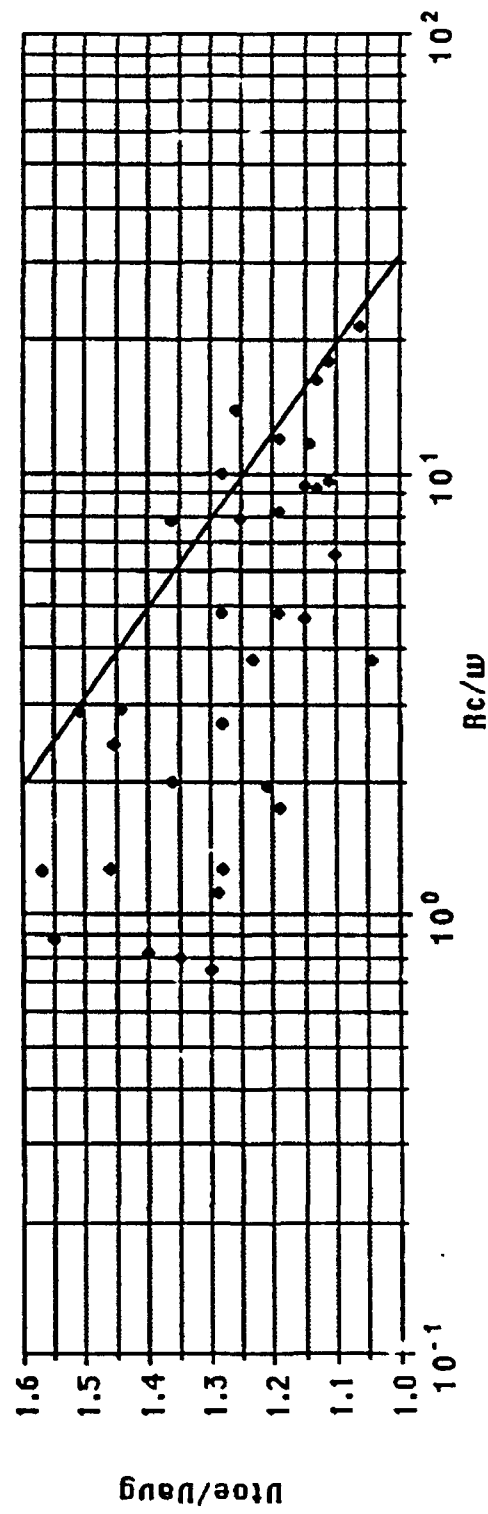


Fig. 3b Trapezoidal Channels

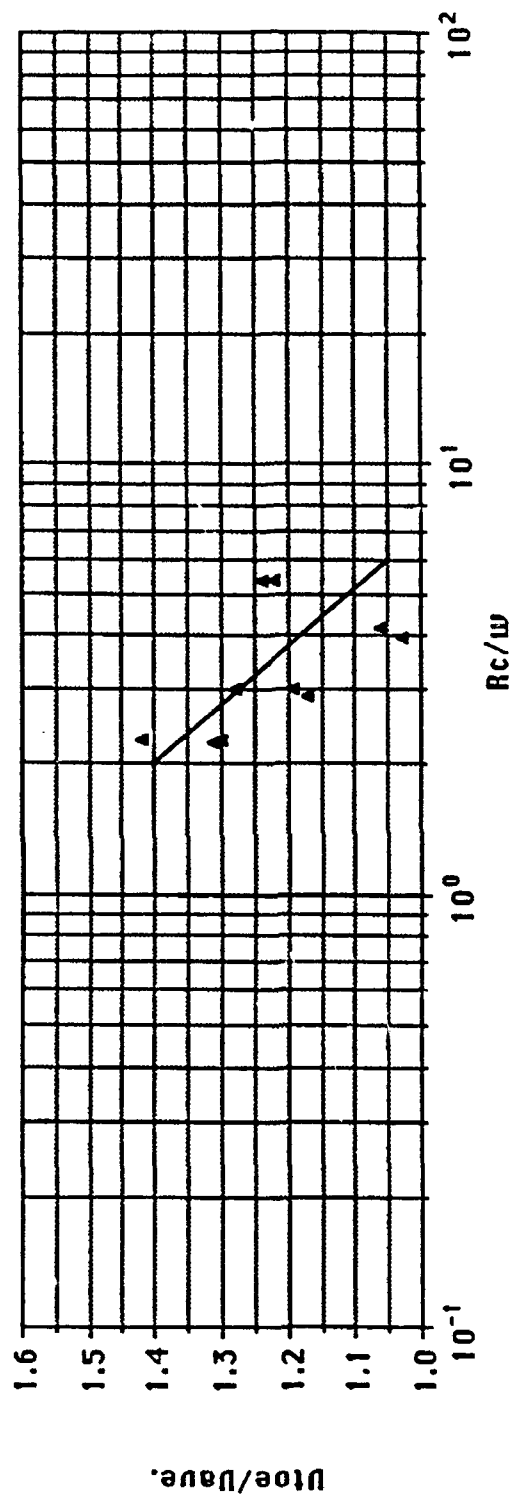
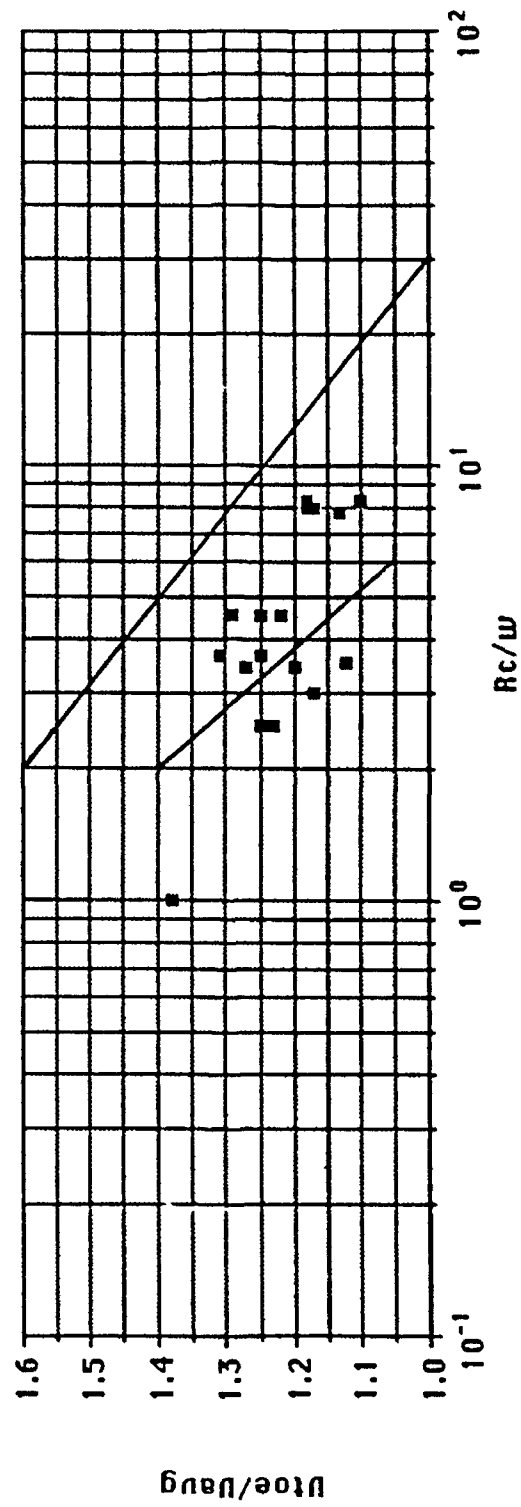


Fig 3c Rectangular Channels



It is concluded that the actual ratio of outer bank toe velocity to average velocity at a bend increases as the ratio of radius of curvature to width decreases, in bends with Rc/w greater than 2. In a natural channel the actual velocity ratio observed in the field is unlikely to exceed the value predicted from the WES design curve, but it is likely to be considerably lower under some circumstances. For very tight bends with Rc/w less than 2, a wide range of V_{toe}/V_{avg} values is possible, but maximum values never exceed 1.6.

Effect of Channel Shape

Figure 3b shows the same plot for trapezoidal channels, again with the relevant WES design curve superimposed. The trend of the line is clearly correct, but the data tend to scatter about the line rather than lying near or below it as in the case of natural channels. Three out of ten points lie significantly above the line, suggesting that it might be prone to underestimating the actual ratio of toe to average velocity under some circumstances.

Figure 3c shows the same plot for rectangular channels. Both the lines for natural and trapezoidal channels are superimposed. The data tend to plot around the line for trapezoidal channels, eleven points lie above and six below the line. As the shape of a rectangular channel is something between trapezoidal and natural, the plotting position of the points is as expected. The plot suggests that V_{toe}/V_{avg} values in rectangular channels are lower than those found in natural channels, but may be somewhat higher than those found in trapezoidal channels.

In order to establish which other variables influence the velocity ratio for a bend, separate semi-logarithmic graphs were plotted for further, different channel characteristics.

Effect of Bank Roughness

Figures 4a and 4b show the Rc/w versus V_{toe}/V_{avg} relations for natural bends with intermediate roughness outer banks and rough outer banks, respectively. Examination of the plots shows complete overlap between the data clouds for the two bank types. This suggests that, for the range of bank roughness represented in the bends studied, the roughness of the outer bank did not materially affect the velocity ratio.

The banks of the laboratory flumes used to generate the data for trapezoidal and rectangular channels showed an insufficient range of roughness to allow separation of the data in this way.

Effect of Bedforms

Figures 5a, b and c show the Rc/w versus V_{toe}/V_{avg} relations for natural bends with plane, ripple and dune, and dune bedforms, respectively. Examination of the plots shows complete overlap of the data clouds for the three bedforms, suggesting that in natural bends the bedform did not significantly affect the velocity ratio.

Fig. 4a Natural Rivers - Intermediate Roughness Outer Banks

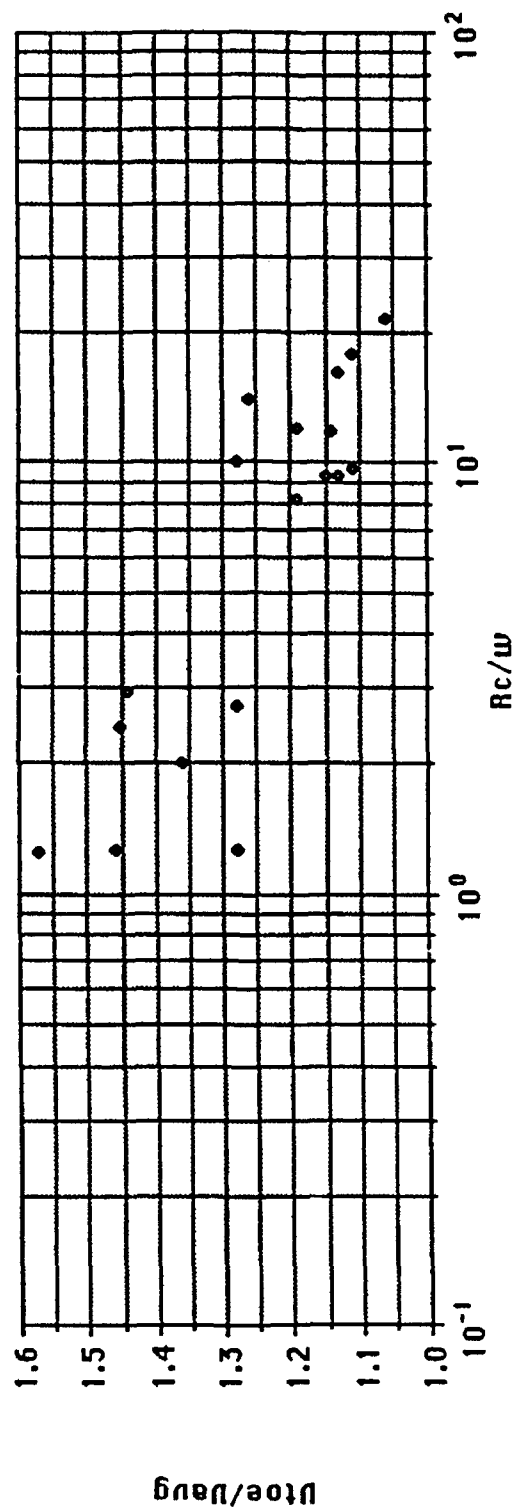


Fig. 4b Natural Rivers - Rough Outer Banks

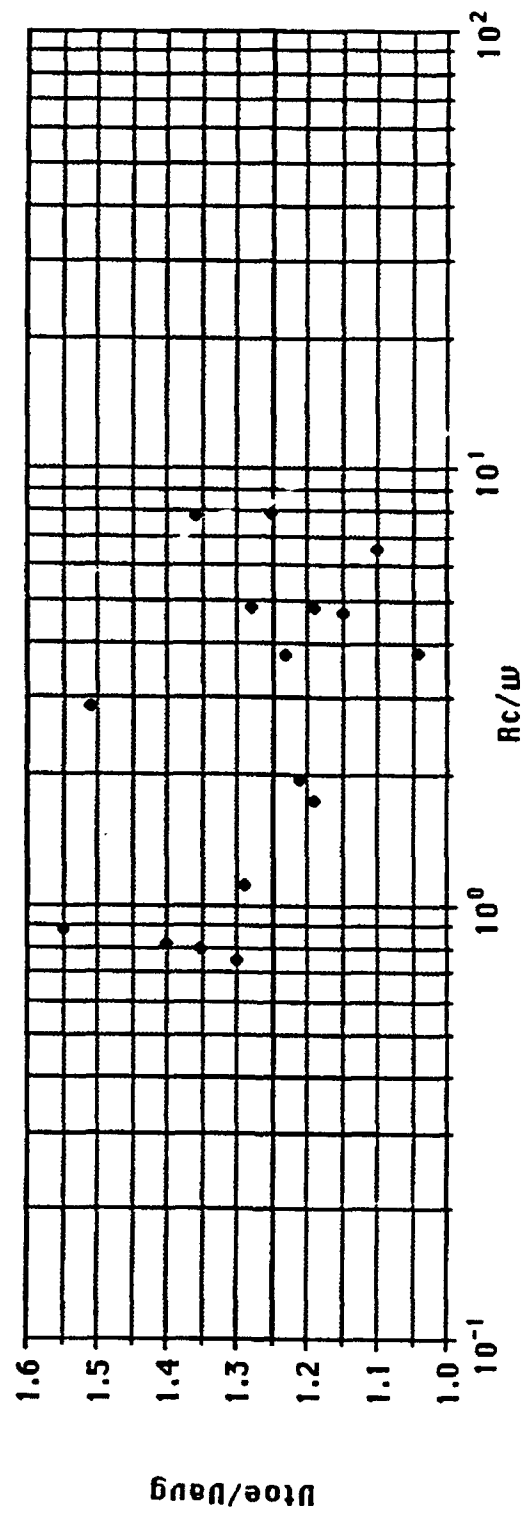


Fig. 5a Natural Rivers - Plane Beds

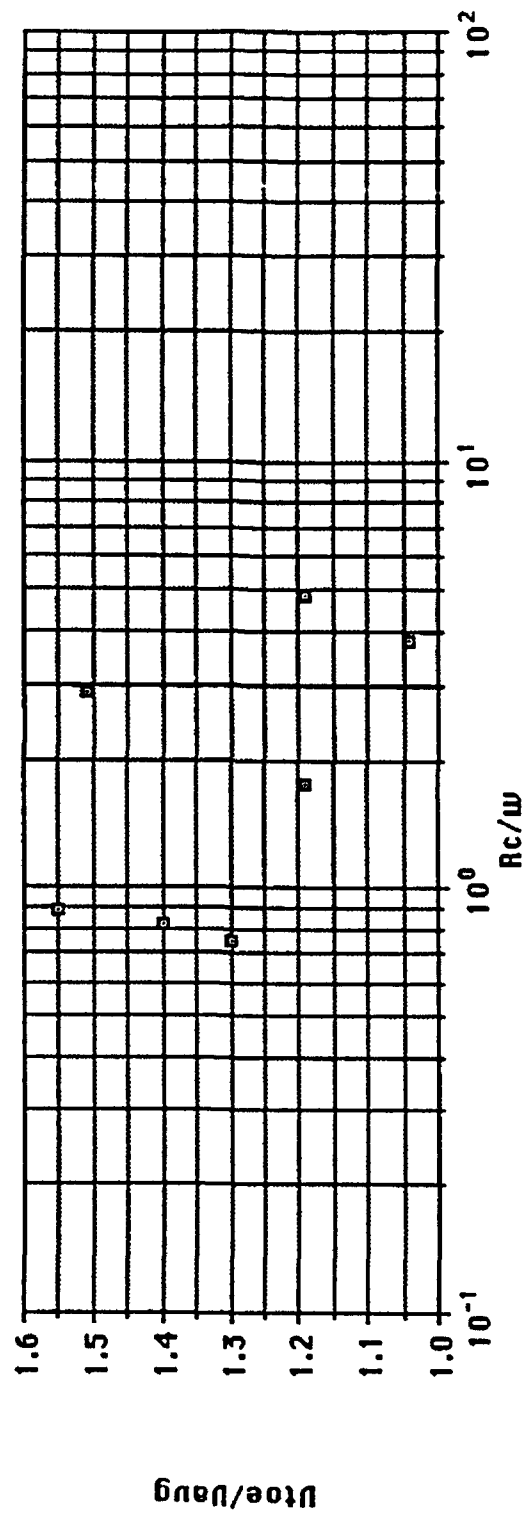


Fig. 5b Natural Rivers - Ripple and Dune Beds

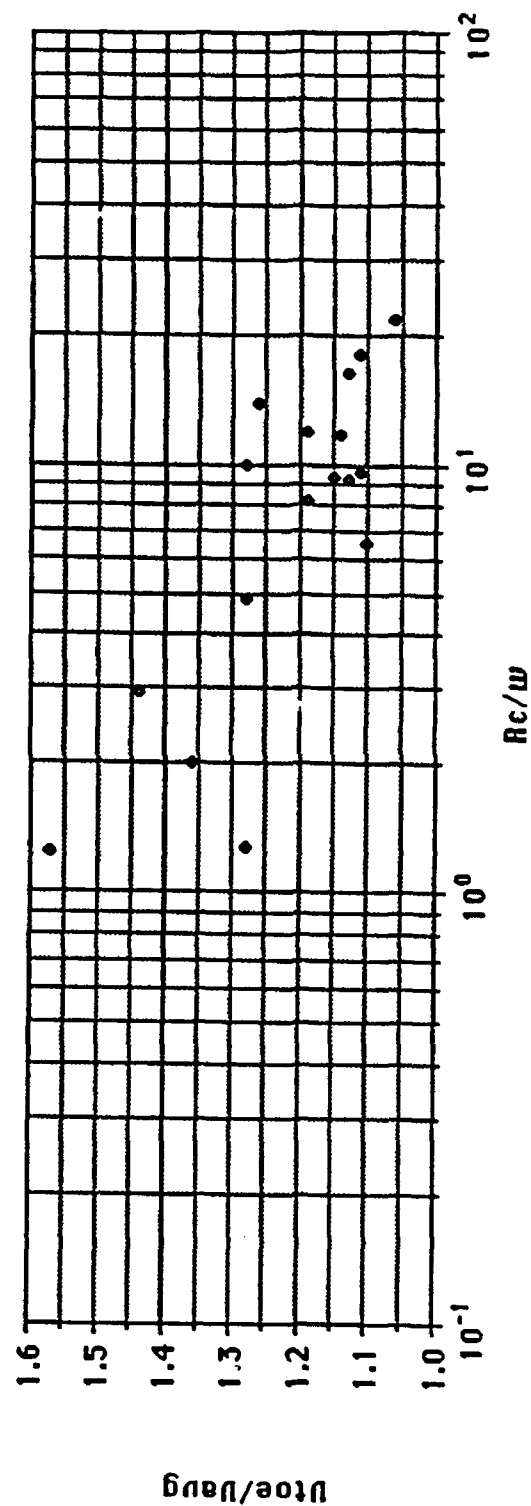
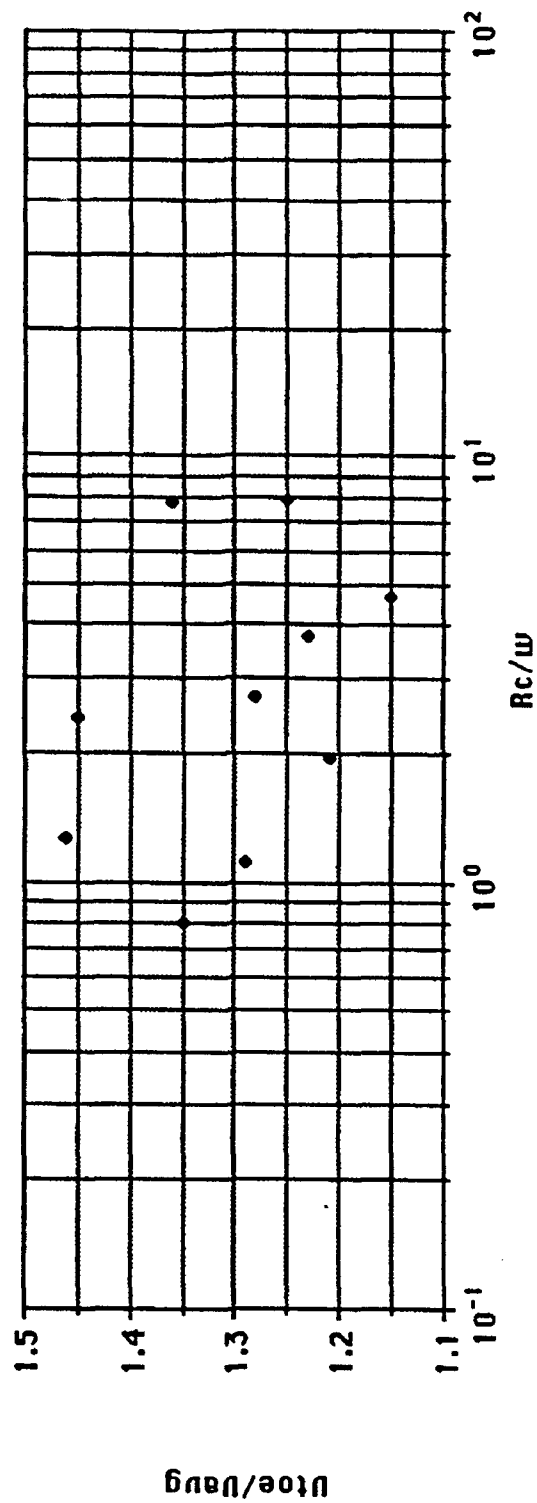


Fig. 5c Natural Rivers - Dune Beds



Figures 6a and b show the relations for rectangular channels with plane and dune beds, respectively. The plots show that dune-bedded bends seem to have higher velocity ratios than plane-bedded bends for the same Rc/w value. This appears to conflict with the finding for natural channels, that bedform did not affect the velocity ratio.

The probable explanation centers on the state of mobility of the bed in the flume studies. The presence of dunes clearly indicates mobile-bed conditions and, therefore, the potential for scour and fill. In a mobile-bed experiment I would expect a scour pool to develop in the bed adjacent to the outer bank and a point bar to form at the inner bank, so that in time the cross-section would come to some extent to resemble that of a natural channel. Data collected at that time would plot between the lines for fixed and natural channels.

Conversely, a plane bed indicates immobile conditions and a fixed, rectangular cross-section. Data from such a channel should plot close to the line for trapezoidal channels. Viewed in this light, the data for different bedforms simply illustrate the effect of cross-sectional shape, with mobile-bed rectangular channels approximating to natural channels and immobile-bed rectangular channels being similar to trapezoidal channels. Re-examination of the data for trapezoidal channels shows that this effect is also evident there. The two points which plot well above the WES design curve both come from a channel with a granular, deformable bed.

On this basis it seems sensible to consider laboratory channels as either having a mobile or an immobile bed, pooling together those with initially trapezoidal and rectangular cross-sections.

Effect of Entrance Conditions

Figures 7a and b show the Rc/w versus V_{toe}/V_{avg} relations for natural channel bends with straight and meandering entrance conditions, respectively. Examination of the plots suggests that for the same Rc/w value, bends downstream of straight reaches have higher velocity ratios than those downstream of meandering reaches. The WES design curve forms a reasonable upper bound to the data for bends with straight entrance conditions, but significantly over-estimates the increase in the velocity ratio that accompanies a decrease in Rc/w for bends in meandering reaches. The discrepancy increases as the Rc/w decreases.

This is, potentially, an important finding because it suggests that different design approaches might be appropriate for single, isolated bends and the consecutive bends of a meandering river.

The difference may arise due to the contrasting transverse distributions of longstream velocity at the entrance of bends with straight and meandering reaches upstream.

At the end of a long, straight approach reach the maximum velocity filament is close to the channel centerline. It must cross only half the channel width before encountering the outer bank zone, and elevating outer bank toe velocities relative to the average velocity. This is usually

Fig. 6a Rectangular Channels - Plane Beds

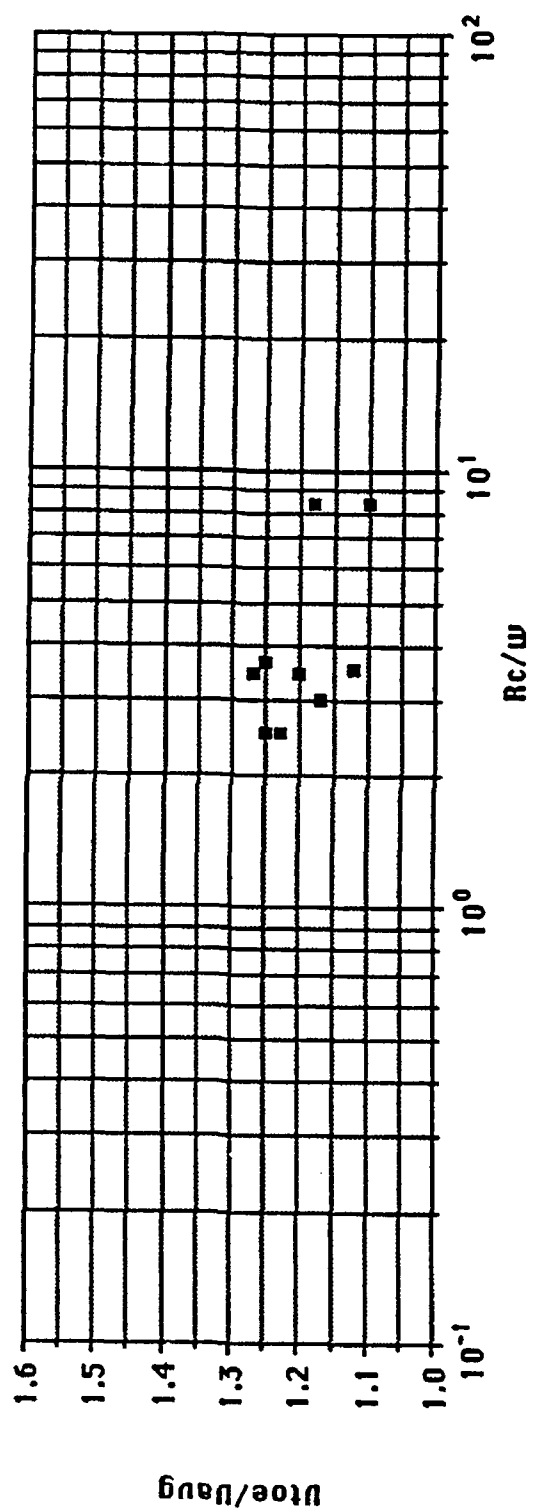


Fig. 6b Rectangular Channels - Dune Beds

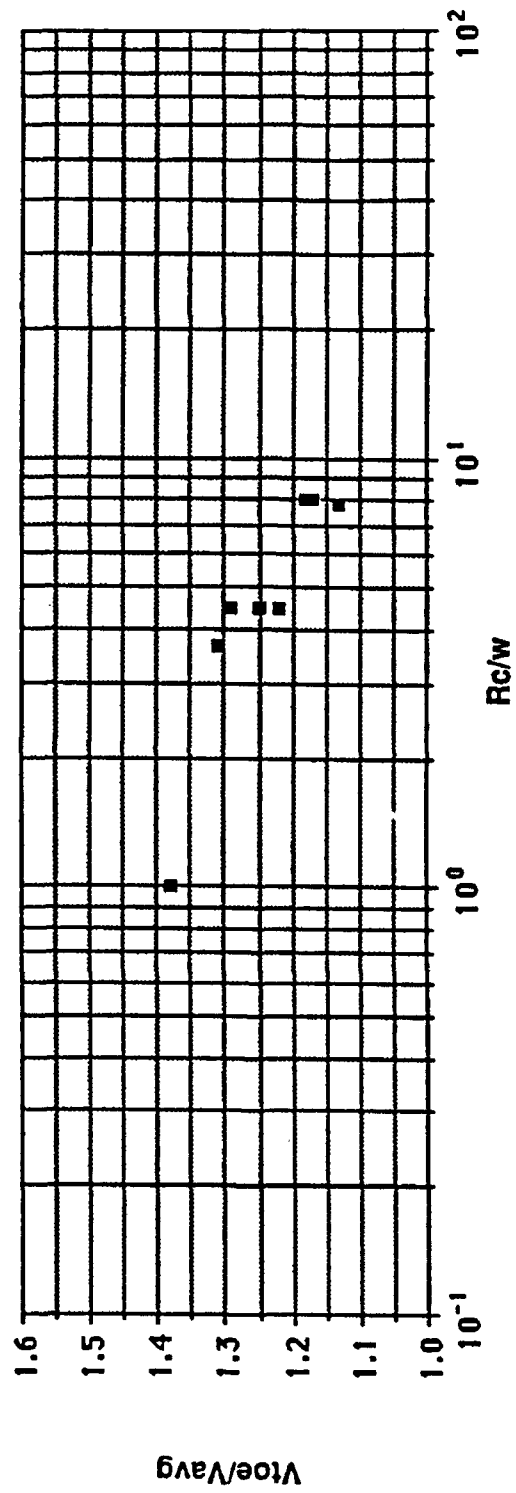


Fig. 7a Natural Rivers - Straight Entrance

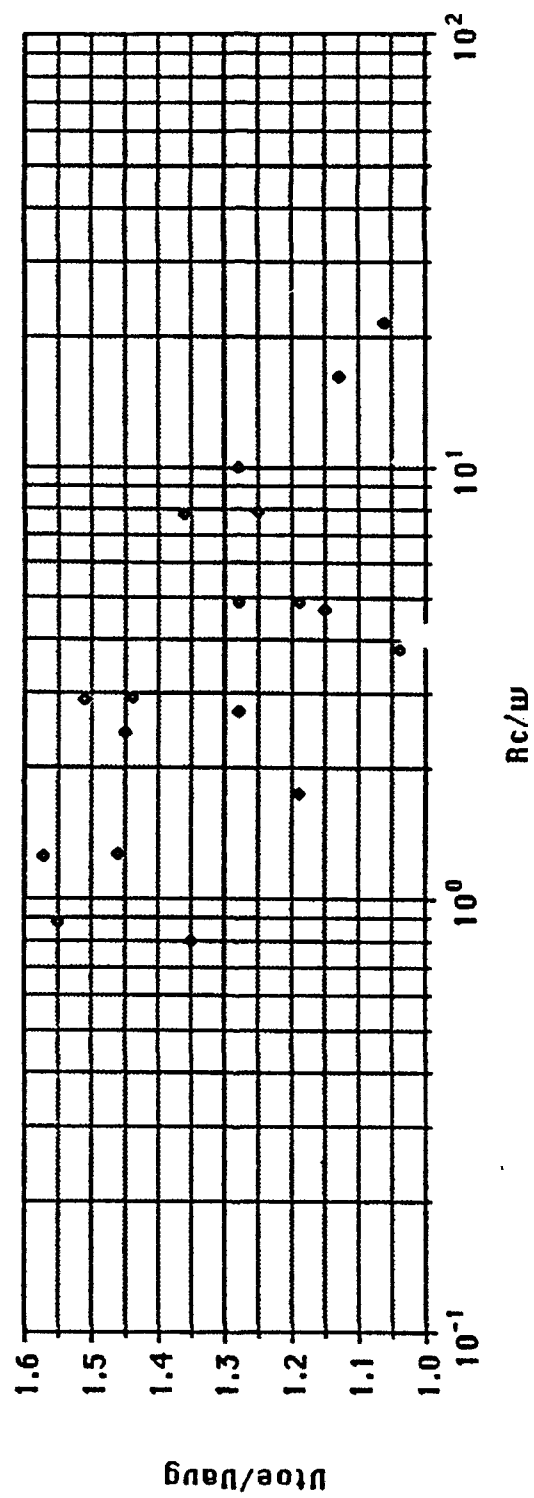
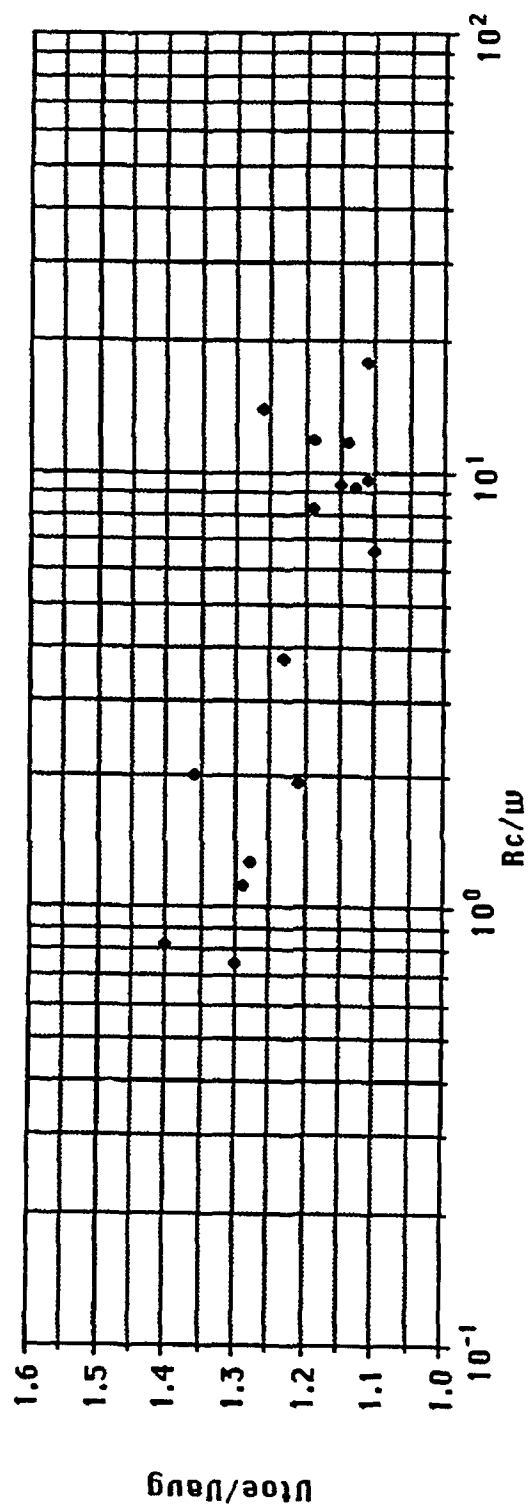


Fig. 7b Natural Rivers - Meandering Entrance



achieved just after the bend apex, in the second half of the bend, where skewing of the flow by the bend is at its strongest.

Conversely, at the exit of an upstream bend the maximum velocity filament is located adjacent to the outer bank for that bend. If the next bend (which is of opposite curvature) is immediately downstream, then at its entrance the maximum velocity filament is located near the inner bank. It must cross the whole channel width before encountering the outer bank zone. This requires fully developed secondary flow and a long bend, and is seldom achieved until downstream of the bend exit, where the strength of skewed flow is already declining. As a result, the ratio of outer bank to average velocity in such a bend may be lower than that in an equivalent bend at the end of a straight reach. This preliminary finding is consistent with long-held ideas on the effect of entrance conditions on bend flow (see for example, Chacinski and Francis, 1952) and it merits further research.

The only break-down of the data to show a systematic and significant difference in the relationship between Rc/w and V_{toe}/V_{avg} was that by entrance condition. It was, therefore, decided to continue this breakdown when examining the impact of the other parameters of bend geometry, and bed and bank characteristics.

Effect of Bend Length

This was investigated using an dimensionless index of bend length (L/w) similar in concept to the dimensionless bend curvature index Rc/w . Figures 8a and b show the relations for natural channel bends with straight and meandering entrance conditions, respectively.

The trend in both graphs is for V_{toe}/V_{avg} to increase as L/w , the dimensionless length of the bend, decreases. This seems surprising since, from first principles, I would have expected that the asymmetry of the flow would be greater in a long bend than a short one due to the greater length of curved channel over which the skewed flow may develop. However, it must be noted that bend length is intimately related to the Rc/w value as well as the L/w value.

It is a geometric fact that tight bends must be short in length, otherwise a neck cut-off occurs. Consequently, there is a high positive correlation ($R = 0.88$) between Rc/w and L/w (Table 10). Hence, short bends have high V_{toe}/V_{avg} ratios not because they are short, but because they are tight. The effects of bend length cannot easily be separated from those of radius of curvature to width ratio. Physically, what happens is that in a short, tight bend the maximum velocity filament crosses the channel rather abruptly and meets the outer bank at an acute angle. The result is that acceleration of the outer bank velocity in a short, tight bend would be expected to be greater than that in a long/gentle bend. The only way to separate the effect of L/w acting alone is through multiple regression, which examines the variation of V_{toe}/V_{avg} caused by a change in L/w after the effects of Rc/w have been accounted for. This analysis is performed later in the report.

Fig. 8a Natural Rivers - Straight Approach Channel

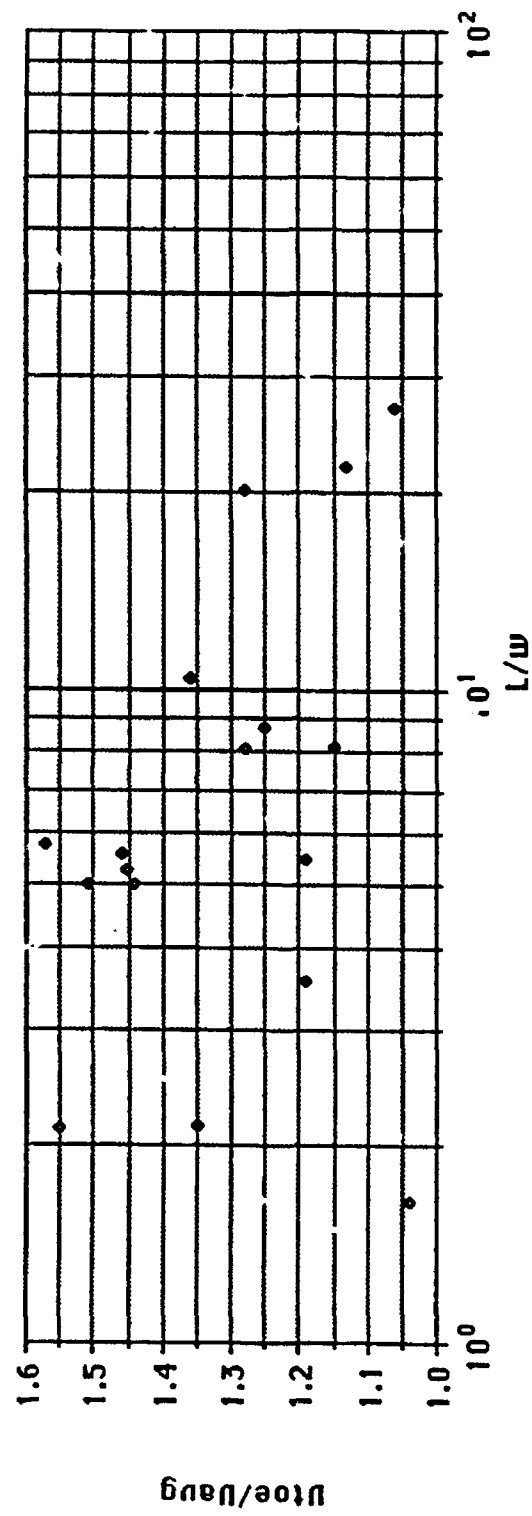


Fig. 8b Natural Rivers - Meandering Approach Channel

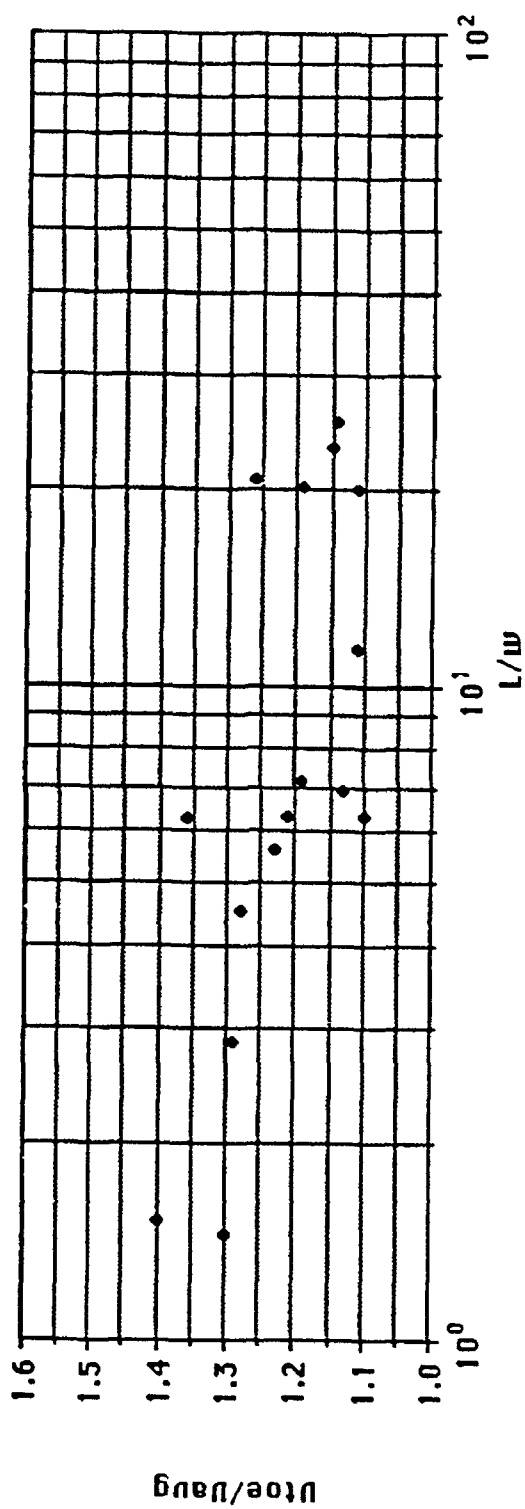


Table 10 Correlation Matrix Between derived Parameters

Correlation Matrix for Variables: $X_1 \dots X_5$					
	Rc/w	L/w	w/d	d/D50	Bank An...
Rc/w	1				
L/w	.881	1			
w/d	.758	.688	1		
d/D50	.858	.834	.885	1	
Bank Angle	-.775	-.777	-.67	-.86	1

The results for trapezoidal and rectangular channels could not be broken down between straight and meandering entrance conditions due to limitations in the scope of the data set. There were not enough cases with meandering entrance conditions to allow a meaningful analysis. As most of the data come from straight approach conditions, the results should be on the safe side with regard to meandering approach channels.

The results are plotted in Figs. 9 and 10 for trapezoidal and rectangular channels, respectively. In both cases the trend of the data is apparently similar to that for natural channels (Fig. 8a and b). However, the comparison of the plots with those for natural channels emphasizes the relatively restricted range of L/w ratios found in the laboratory channels, which makes it less easy to identify any trend in the data.

Effect of Aspect Ratio

Figures 11a and b show the relations for aspect ratio (w/d) versus V_{toe}/V_{avg} in natural channel bends with straight and meandering entrance conditions, respectively.

The trend in both graphs is for V_{toe}/V_{avg} to increase as width to depth ratio decreases. This is explained by the different flow patterns in narrow, deep channels versus shallow, wide channels. In a shallow, wide channel bed roughness dominates the flow pattern, flow is pseudo two-dimensional, and both curvature and outer bank effects are relatively less important. Conversely, in a narrow, deep channel the flow is fully three-dimensional and strong skew-induced and outer bank secondary currents dominate the flow pattern. It is these secondary currents which are responsible for the acceleration of near bank primary velocities in the outer bank region. Consequently, the increase in the outer bank toe velocity is much greater in a narrow, deep channel than in an equivalent shallow, wide one. Again, Table 7 reveals a positive correlation between w/d and Rc/w ($R = 0.76$), however. Therefore, final consideration of the effect of w/d on V_{toe}/V_{avg} must be reserved until a multiple regression has been performed.

Fig. 9 TRAPEZOIDAL CHANNELS

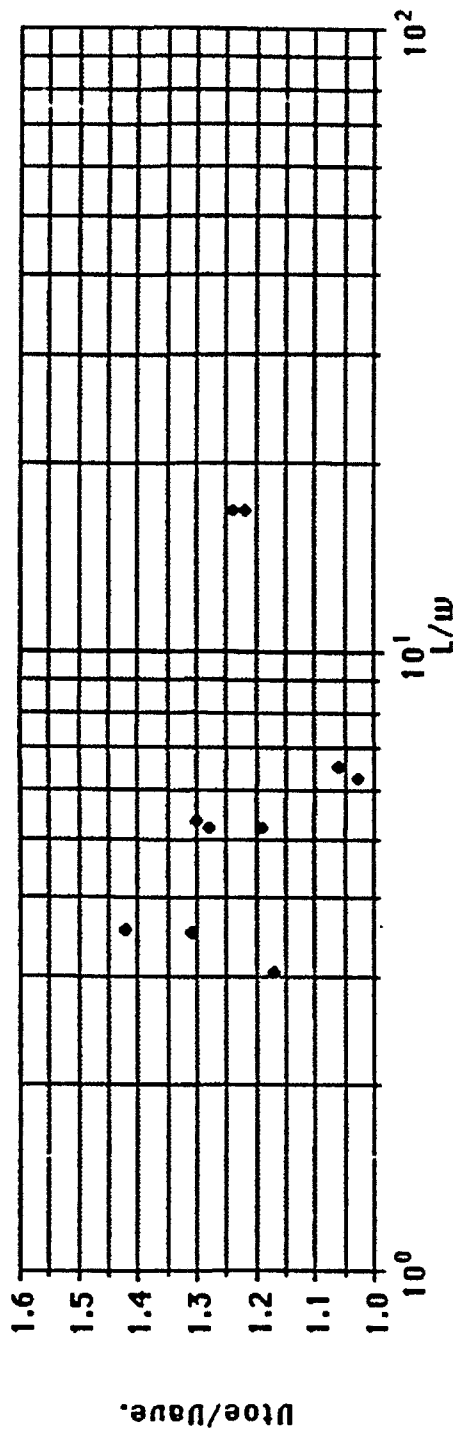


Fig. 10 Rectangular Channels

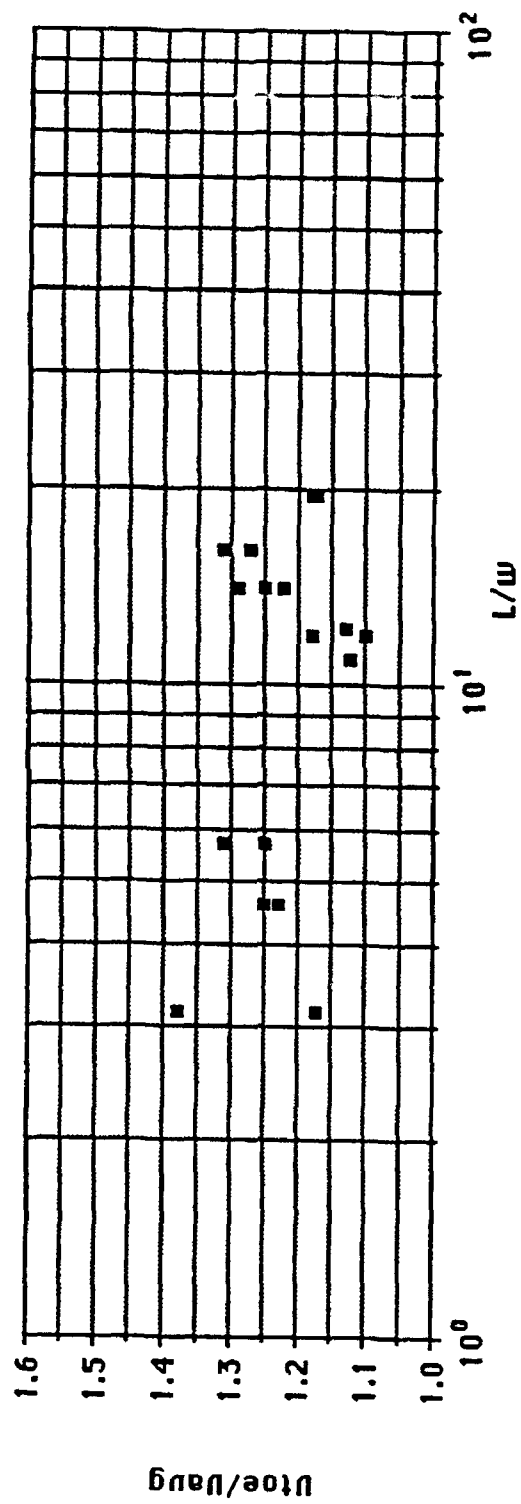


Fig. 11a Natural Rivers - Straight Approach Channel

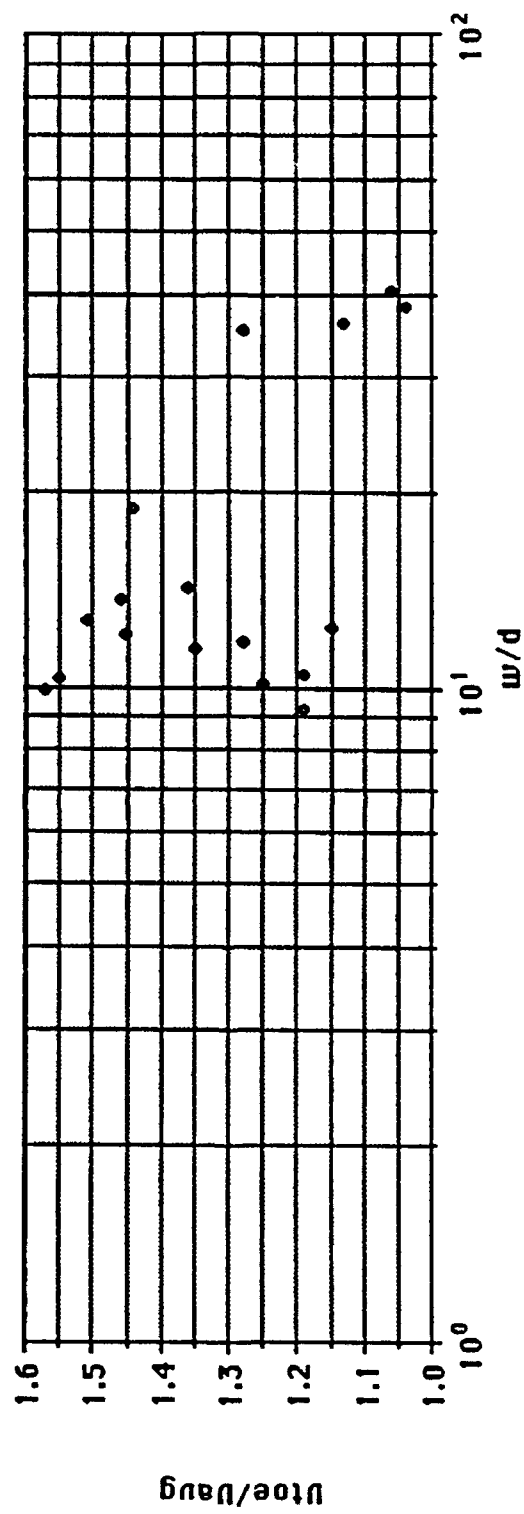
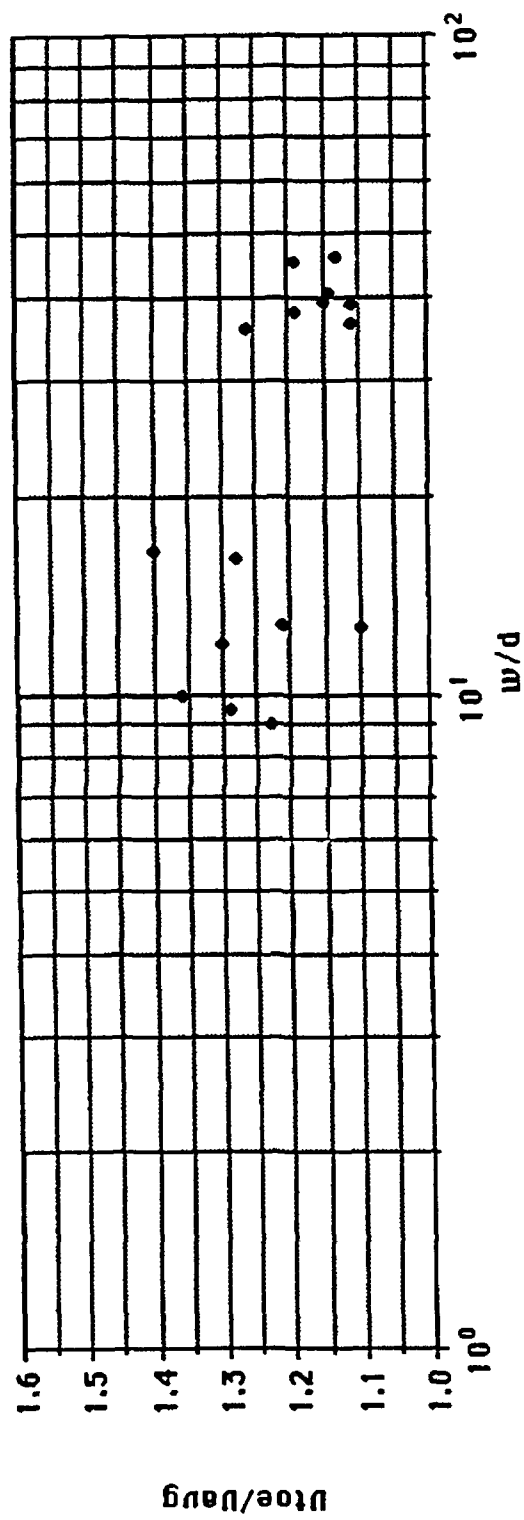


Fig. 11b Natural Rivers - Meandering Approach Channel



The results for trapezoidal and rectangular channels are shown in Figs. 12 and 13, respectively. The ranges of w/d encountered in laboratory channels are severely restricted compared to natural channels, but the trend in the data does not contradict the results for natural channels.

Effect of Relative Depth

Figures 14a and b show the relations for relative depth ($d/D50$) versus V_{toe}/V_{avg} in natural channel bends with straight and meandering entrance conditions, respectively.

The trend in both graphs is for V_{toe}/V_{avg} to increase as relative depth decreases. This is explained by the effect of flow resistance on average velocity in a channel. Relative depth (the inverse of relative roughness ($D50/d$)) characterizes how deep the flow is in terms of the median size of the bed material. It represents the hydraulic smoothness of the channel, which is primarily responsible for determining the flow resistance coefficient, and hence the average velocity. In a relatively deep, smooth channel (high $d/D50$) resistance is low and average velocity high when compared to an otherwise equivalent shallow, rough channel (low $d/D50$). Consequently, when the outer bank toe velocity is divided by that average velocity in a deep, smooth channel, the quotient is lower than in an otherwise equivalent shallow, rough channel. The difference becomes more marked as $d/D50$ decreases. It should be noted though that this effect operates on the quotient V_{toe}/V_{avg} . It does not mean that absolute values of V_{toe} will be lower in a deep, smooth channel than in a shallow, rough one, because the actual value of V_{avg} may also be much higher in the smoother channel. The correlation matrix (Table 7) shows a strong, positive correlation between $d/D50$ and R_c/w ($R = 0.86$). Hence, final judgement on the relationship between $d/D50$ and V_{toe}/V_{avg} must await the multiple regression analysis.

There is considerable scatter in both the graphs, especially that for straight approach conditions (Fig. 14a), but the points for bends with straight entrance approach channels consistently plot higher than those for meandering approaches. This suggests that compared to meandering rivers, maximum values of V_{toe}/V_{avg} are higher for bends with straight approach channels, and increase more markedly as $d/D50$ decreases.

The results for trapezoidal and rectangular channels are shown in Figs. 15 and 16, respectively. The trends in the data support the results for natural channels over a wide range of relative depths.

Effect of Outer Bank Angle

Figures 17a and b show the relations for outer bank angle versus V_{toe}/V_{avg} in natural channel bends with straight and meandering entrance conditions, respectively.

The trend in both graphs is for V_{toe}/V_{avg} to increase with increasing bank angle.

Fig. 12 TRAPEZOIDAL CHANNELS

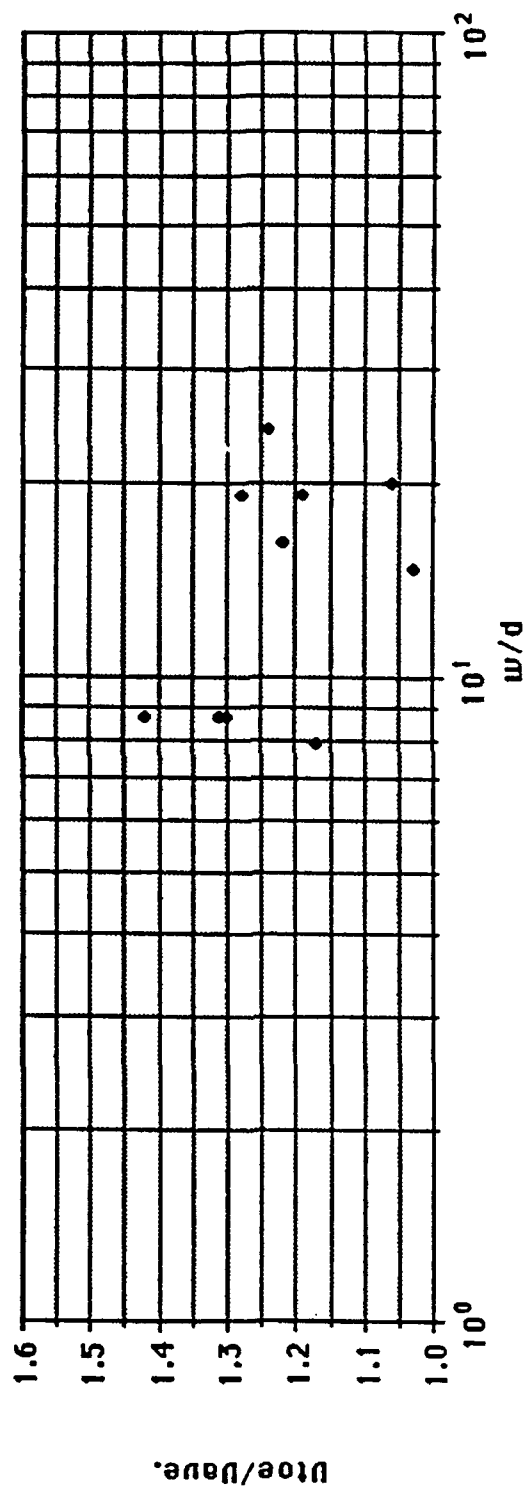


Fig. 13 Rectangular Channels

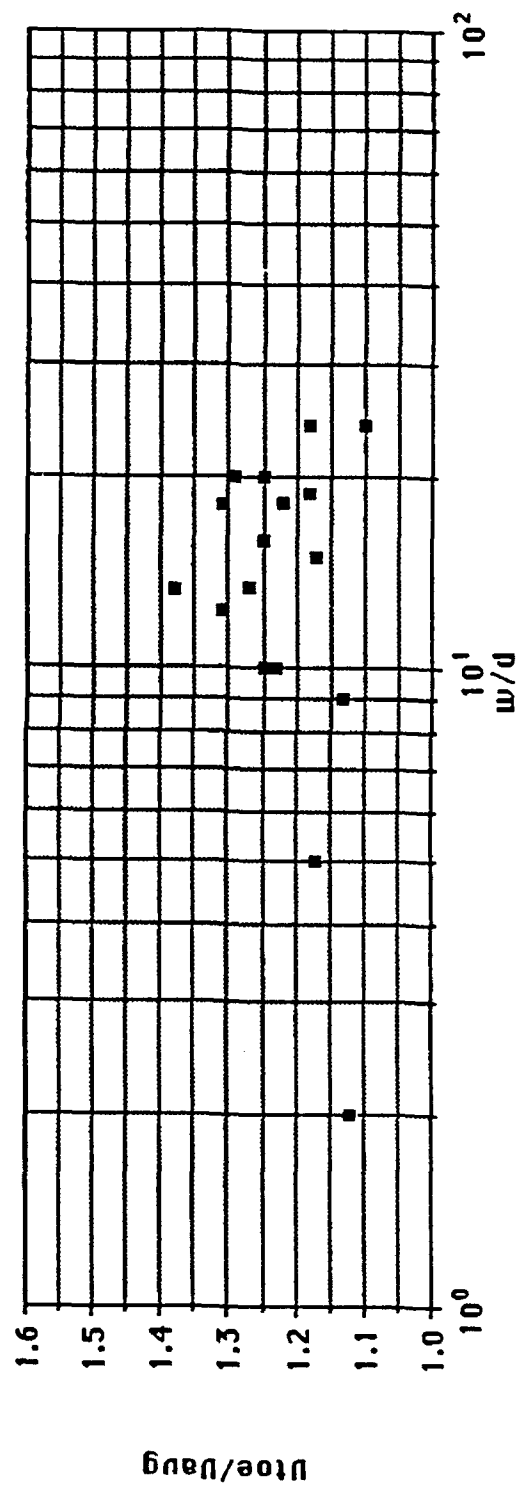


Fig. 14a Natural Rivers - Straight Approach Channel

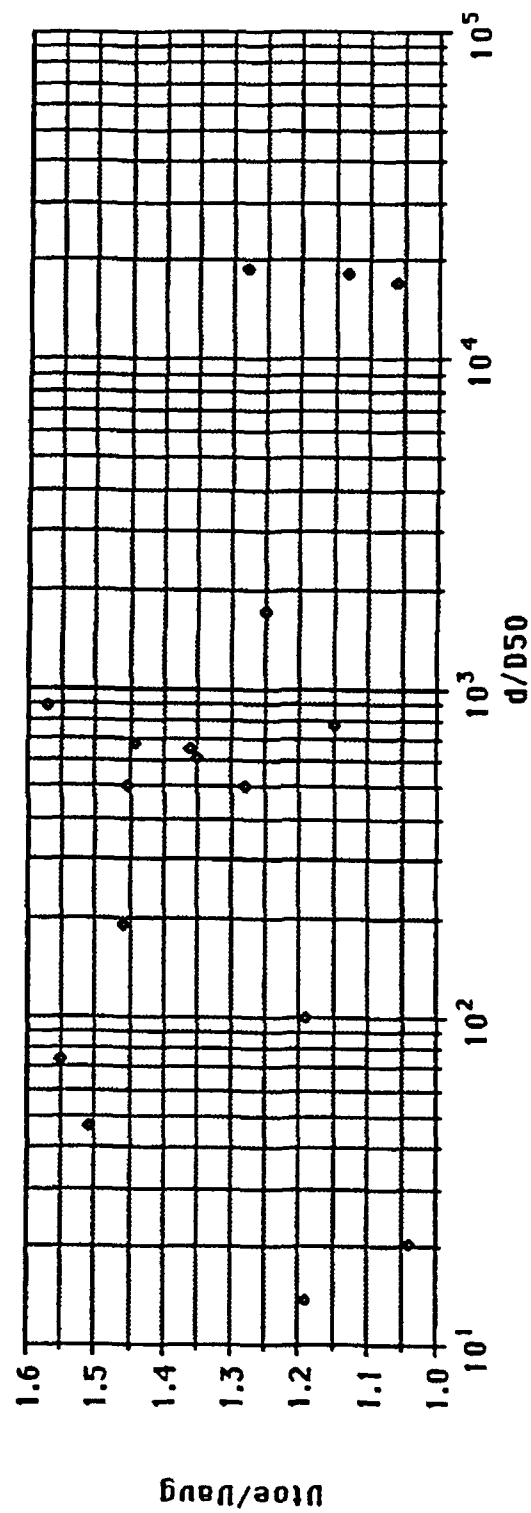


Fig. 14b Natural Rivers - Meandering Approach Channel

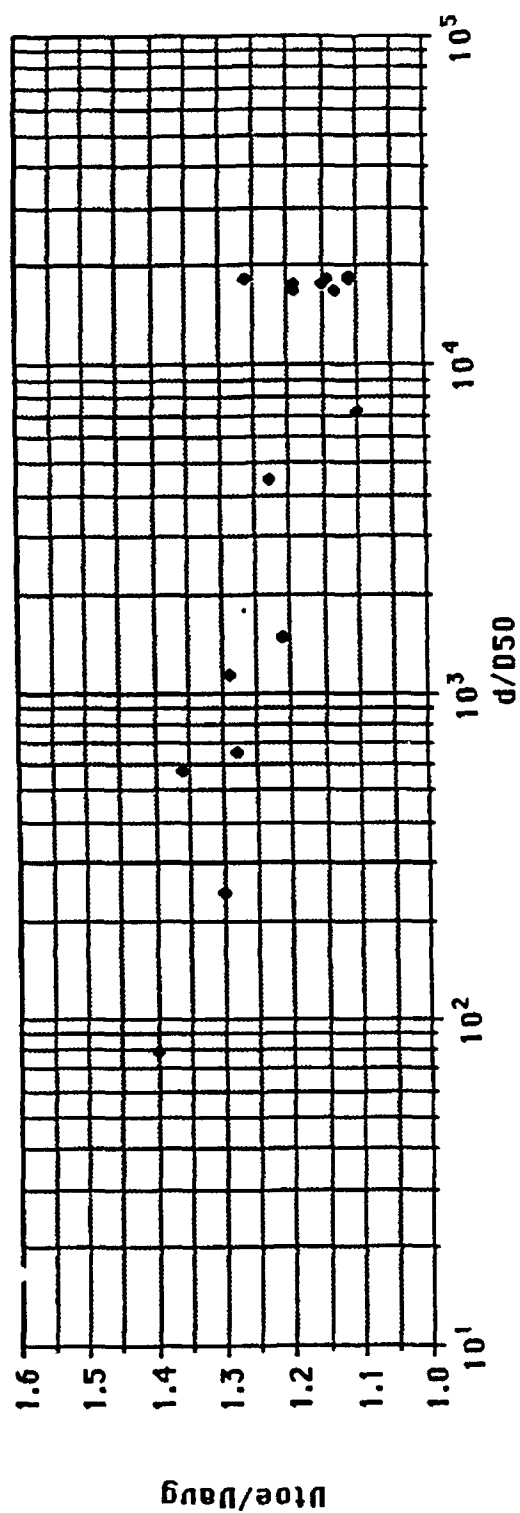


Fig. 15 TRAPEZOIDAL CHANNELS

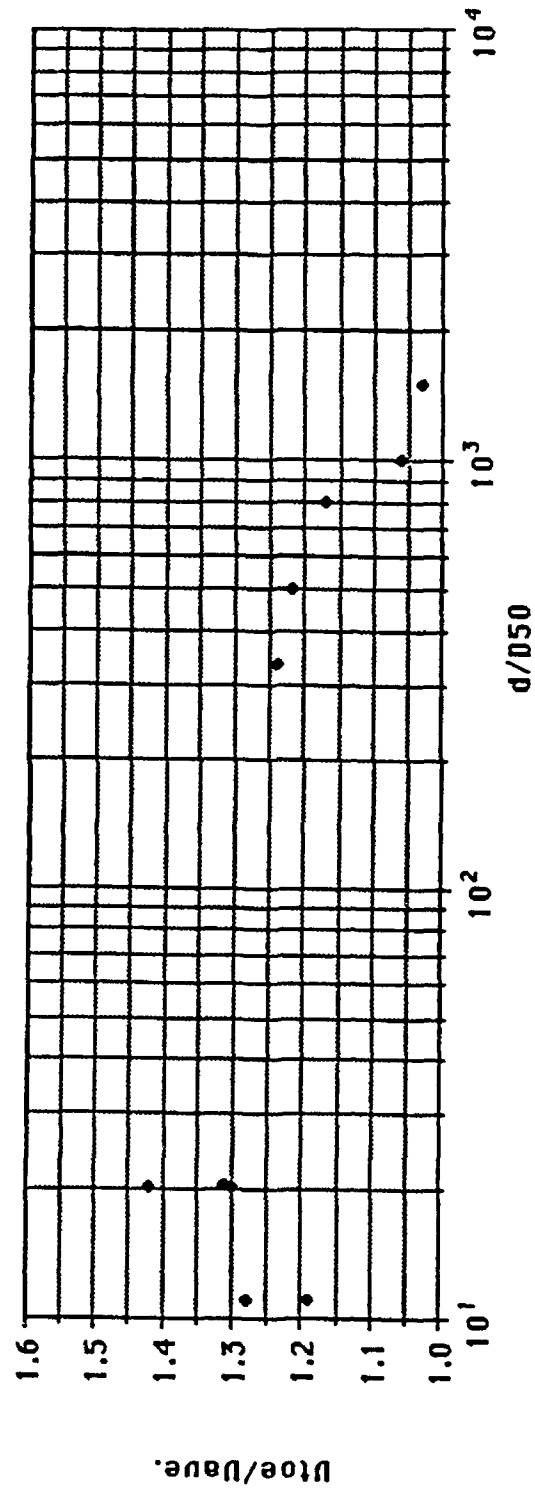


Fig. 16 Rectangular Channels

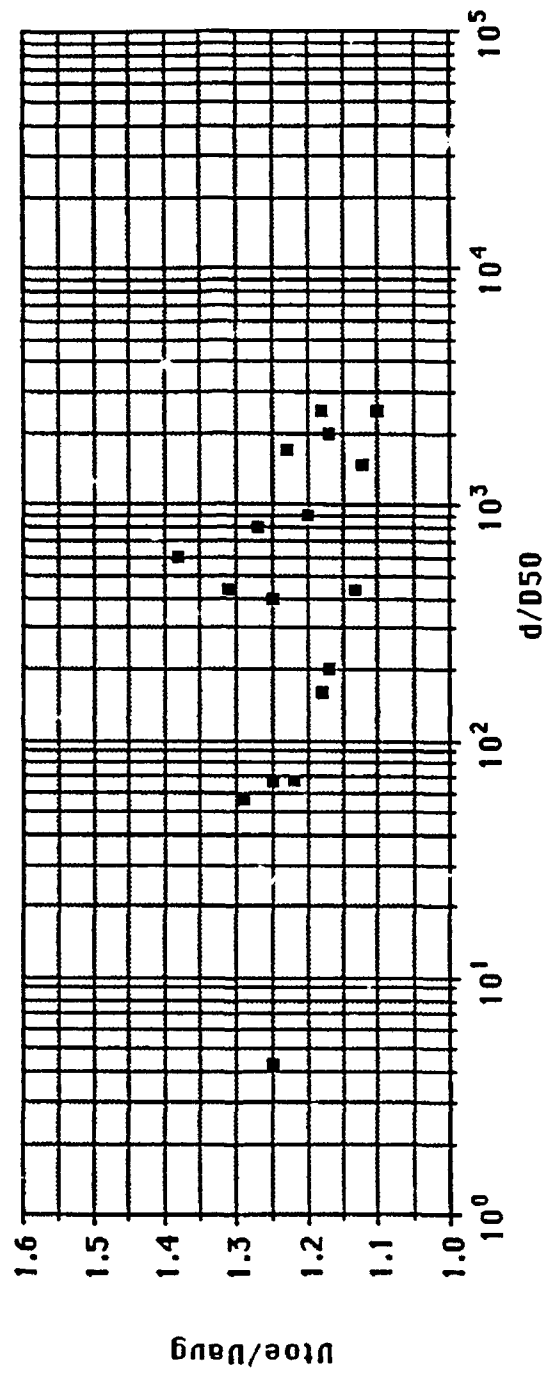


Fig. 17a Natural Rivers - Straight Approach Channel

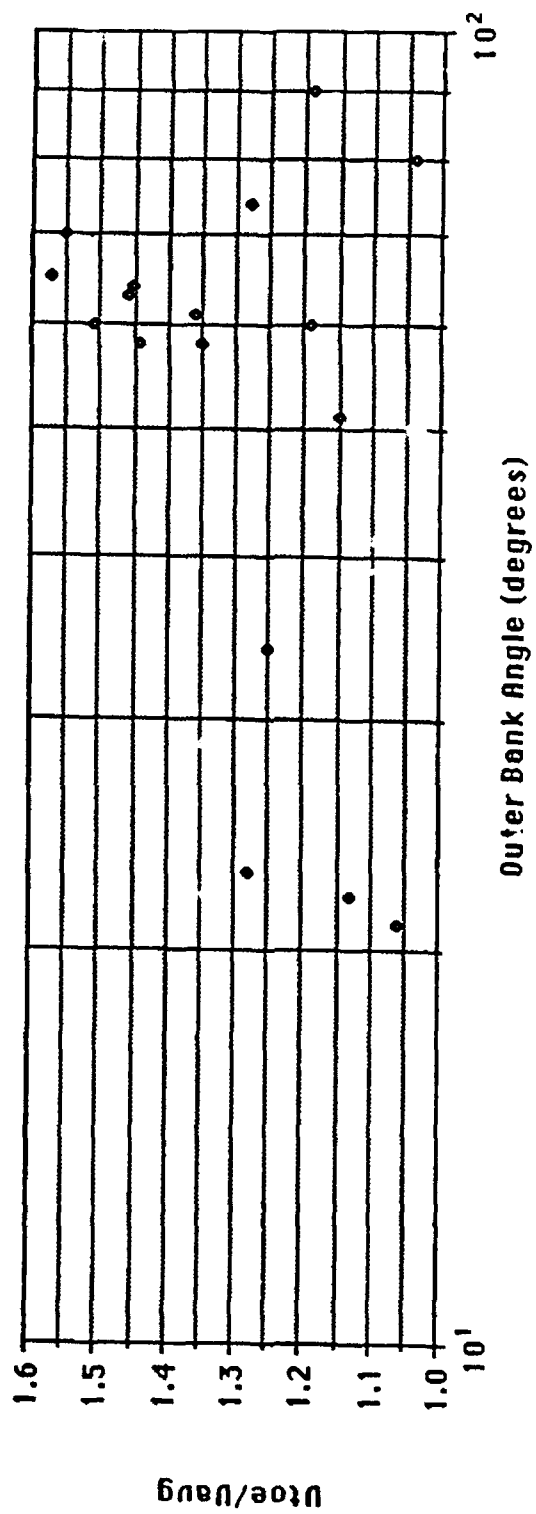
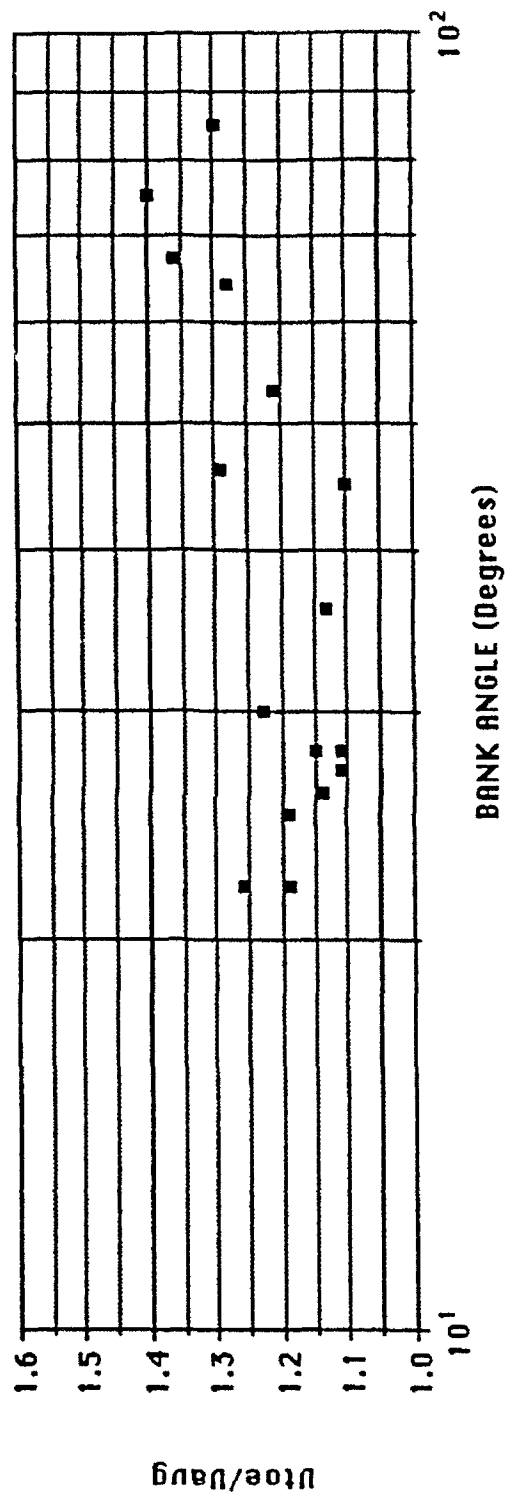


Fig. 17b Natural Rivers - Meandering Approach Channel



There is weak evidence that the V_{toe}/V_{avg} ratio might level off, or even decrease again as bank angle increases from about 75 to 90 degrees. The increase in V_{toe}/V_{avg} with increasing bank angle is explained by the effect of outer bank angle on flow patterns adjacent to the bank. In a channel with a shelving outer bank, depth decreases gradually as the water's edge is approached. Relative roughness (D_{50}/d) increases to high values, and momentum (both longstream and cross-stream) is lost to friction at the boundary. The high roughness, low depth and low velocities combine to suppress all but the strongest secondary currents, and no outer bank cell of secondary circulation develops. Hence, the maximum primary velocity in the bank zone is relatively low, and it stays at the free surface, well away from the toe.

But in a bank with a steep outer bank, the depth increases quickly at the water's edge. High velocity flow is able to move in close to the bank, and secondary currents are pronounced. The junction of the water surface and the bank acts as a stagnation point where the velocity of outward flow near the surface must go to zero very abruptly as the skew-induced cell meets the outer bank. This results in a small cell of reverse rotation. Combination of the outer bank and skew-induced cells with the high velocity primary flow, produces a depressed maximum in the region above the bank toe, which can produce much higher V_{toe}/V_{avg} values than those found adjacent to a shelving, low-angle bank.

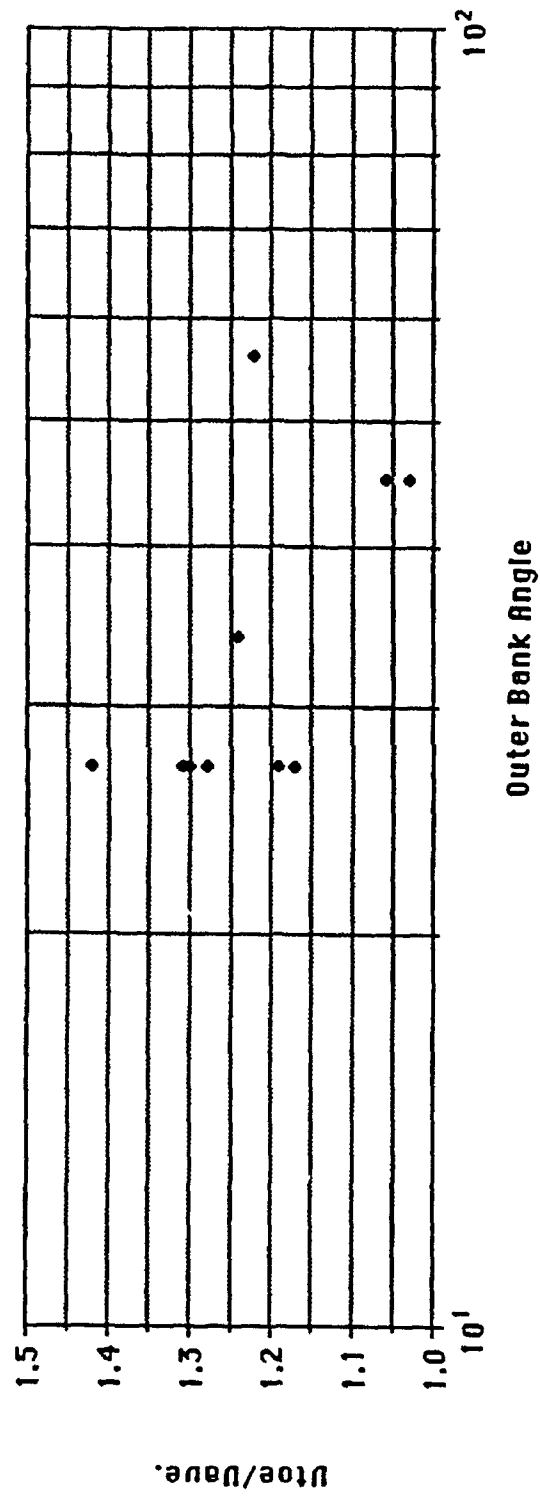
The apparent reduction in V_{toe}/V_{avg} observed in bends with near vertical outer banks probably results from the definition of V_{toe} used here. This is the depth-averaged longstream velocity over the toe of the bank. But, when the bank angle is close to 90° , this definition breaks down because the bank toe and bank top practically coincide. That is, the V_{toe} corresponds to the velocity at the water's edge, which is relatively reduced by simple boundary friction compared to that a finite distance out into the channel. Also, the maximum near bank velocity is located about one third of the way up the bank, rather than at the toe. In such cases, a modified parameter is needed to represent the characteristic velocity adjacent to the outer bank.

The correlation between bank angle and R_c/w ($R = -0.78$) is a strong, negative one (Table 10). This indicates that steeper banks are encountered in tighter bends. Therefore when considering the effect of bank angle on V_{toe}/V_{avg} , final conclusions cannot be drawn until after completion of the multiple regression analysis.

The scatter of points for straight versus meandering entrance conditions shows that the distributions overlap for low bank angles, but that the increase in V_{toe}/V_{avg} with increased bank angle appears more marked in channels with a straight approach.

The data trapezoidal channel are shown in Fig. 18. Although sparse, they do not contradict the results for natural channels. Rectangular channels have vertical outer banks by definition and so no plot could be generated for this variable.

Fig. 18 TRAPEZOIDAL CHANNELS



Main Points

After this preliminary examination of the data the following points were noted:

1. The WES design curve represents a conservative approach to the estimation of (V_{toe}/V_{avg}) in natural channels.
2. Points for bends of very low Rc/w values reveal that the monotonic increase in V_{toe}/V_{avg} observed as Rc/w decreases may cease at an Rc/w of about 2.
3. For trapezoidal channels, the WES design curve might be prone to underestimating the actual ratio of toe to average velocity under some circumstances.
4. V_{toe}/V_{avg} values in immobile bed rectangular channels are similar to those found in immobile bed trapezoidal channels.
5. The roughness of the outer bank does not significantly affect the ratio of outer bank toe velocity to average velocity in the bends studied.
6. The presence of bedforms does not significantly affect the ratio of outer bank toe velocity to average velocity in the bends studied. It is noted, though, that the velocity ratio for immobile, trapezoidal or rectangular channels is significantly lower than that for mobile-bed, trapezoidal or rectangular channels with the same Rc/w ratio.
7. The ratio of outer bank toe velocity to average velocity in a bend immediately downstream of a straight reach may be significantly higher than that in a bend downstream of a bend of opposite curvature. The WES design line appears to be a good upper bound for bends downstream of straight reaches but may over-estimate the velocity ratio in bends in meandering reaches.
8. Bend geometry parameters L/w , w/d and $d/D50$ are all show strong, positive correlations with Rc/w . This makes it difficult to identify their individual effects on V_{toe}/V_{avg} . From the individual plots it appears that V_{toe}/V_{avg} increases as L/w , w/d and $d/D50$ decrease.
9. Bank angle, α , shows a strong, negative correlation with Rc/w . V_{toe}/V_{avg} appears to increase as the bank angle increases. There is some evidence that V_{toe}/V_{avg} might decrease as bank angle increases from about 75 to 90 degrees.

It should be noted that there is considerable scatter in the graphs produced here. It would be a simple matter to "massage" the data in order to obtain tighter distributions. This temptation has been resisted, however, and the data are exactly as they were extracted from the various reports and papers. No doubt, some errors of judgement have been made and some bends have been mis-classified. But re-assigning bends to different "straight" and "meandering" categories now would undermine the objectivity of the study. The analysis is based on the our best judgement without the benefit of hind-sight. If it cannot produce good results under these circumstances, then it would be unlikely to be of practical use.

Analysis of Data

The examination of the data has established that variables other than Rc/w appear to influence the ratio of outer bank toe velocity to average velocity in a channel bend. Up to now, each of the potentially significant variables has been considered separately, as if it acted alone to influence V_{toe}/V_{avg} . In nature this is not the case, as the actual V_{toe}/V_{avg} value for a bend is the product of the mutual interaction of all the significant controlling variables. Significant correlations exist between the variables, and these may obscure the true effect of each on the dependent variable (V_{toe}/V_{avg}).

To develop a predictive approach which reflects this situation, multiple regression was used to produce equations expressing the dependent variable V_{toe}/V_{avg} as a function of all "independent" variables identified as being potentially significant in the previous section. A stepwise regression was used. This adds each independent variable in turn to the regression analysis, rejecting any variable which does not add significantly to the strength of the regression equation.

From the examination of the data it was concluded that the linear relation between Rc/w and V_{toe}/V_{avg} may break down for Rc/w values less than 2. There are sound theoretical reasons to expect this, and the data are consistent with the idea that as bends tighten to Rc/w values less than 2 major changes in flow pattern occur, often leading to impinging flow and areas of separation at the outer bank. In view of this, it was decided to limit the curve fitting to bends with Rc/w values equal to or greater than 2.

It was noted earlier that in some studies only a few sections were monitored in each bend, and that consequently the data collected do not represent the absolute maximum ratio of V_{toe}/V_{avg} for that bend. This explains why some points plot low in the various distributions shown in Fig. 3. Taking this fact together with the recommendation from WES that an upper bound line is preferable to a best-fit line when predicting V_{toe}/V_{avg} for riprap design, it was decided to perform the multiple regression using only points from the top edge of the scatter. These turned out to come mostly from bends which were intensively studied (for example, studies by Thorne et al., Dietrich, Bridge). Some data came from single sections too, apparently where those sections happened to have coincided with the highest values of V_{toe}/V_{avg} for the bend. Hence, the correlation coefficients indicate the linearity of the upper surface of the data cloud, rather than the strength of the regression as such.

Prediction of Outer Bank Velocity in Open Channel Bendways

Natural Rivers

In line with the finding that the approach condition did appear to affect the velocity ratio for natural rivers, separate analyses were performed for straight and meandering approach conditions.

Natural Rivers: Straight Approach Conditions

The points used to define the upper boundary are indicated in Fig. 19. The equation for natural rivers with straight entrance conditions is:

$$\frac{V_{TOE}}{V_{AVG}} = 1.29 - 0.5 \log \left(\frac{Rc}{w} \right) + 0.4 \log \left(\frac{L}{w} \right) - 0.06 \log \left(\frac{d}{D_{50}} \right) + 0.16 \log (a) \quad (5)$$

The adjusted coefficient of determination for this equation (r^2) is 0.905, indicating that the equation is well fitted to the upper surface of the data cloud. The width-depth ratio did not contribute sufficiently to appear in the equation.

For comparison with the WES design curve, and to evaluate whether the improvement in accuracy merits the increased complexity of eqn. 5, a line was also fitted to the data using simple regression for V_{toe}/V_{avg} as a function of Rc/w . The resulting equation is:

$$\frac{V_{TOE}}{V_{AVG}} = 1.66 - 0.42 \log \left(\frac{Rc}{w} \right) \quad (6)$$

The adjusted coefficient of determination for this equation (r^2) is 0.90, indicating good linearity at the upper edge of the data cloud. The resulting line is shown in Fig. 19.

Natural Channels: Meandering Approach Conditions

The points used to define the upper boundary are shown in Fig. 20. The resulting equation for natural rivers with meandering entrance conditions is:

$$\frac{V_{TOE}}{V_{AVG}} = 0.95 - 0.16 \log \left(\frac{Rc}{w} \right) - 0.06 \log \left(\frac{L}{w} \right) + 0.06 \log \left(\frac{w}{d} \right) + 0.25 \log (a) \quad (7)$$

The adjusted coefficient of determination for this equation (r^2) is 0.985, indicating that the equation is well fitted to the upper surface of the data cloud. The relative depth did not contribute sufficiently to appear in the equation.

For comparison with the WES design curve, and to evaluate whether the improvement in accuracy merits the increased complexity of eqn. 7, a line was also fitted to the data using simple regression for V_{toe}/V_{avg} as a function of Rc/w . The resulting equation is:

$$\frac{V_{TOE}}{V_{AVG}} = 1.4 - 0.24 \log \left(\frac{Rc}{w} \right) \quad (8)$$

Fig. 19 Natural Rivers - Straight Entrance

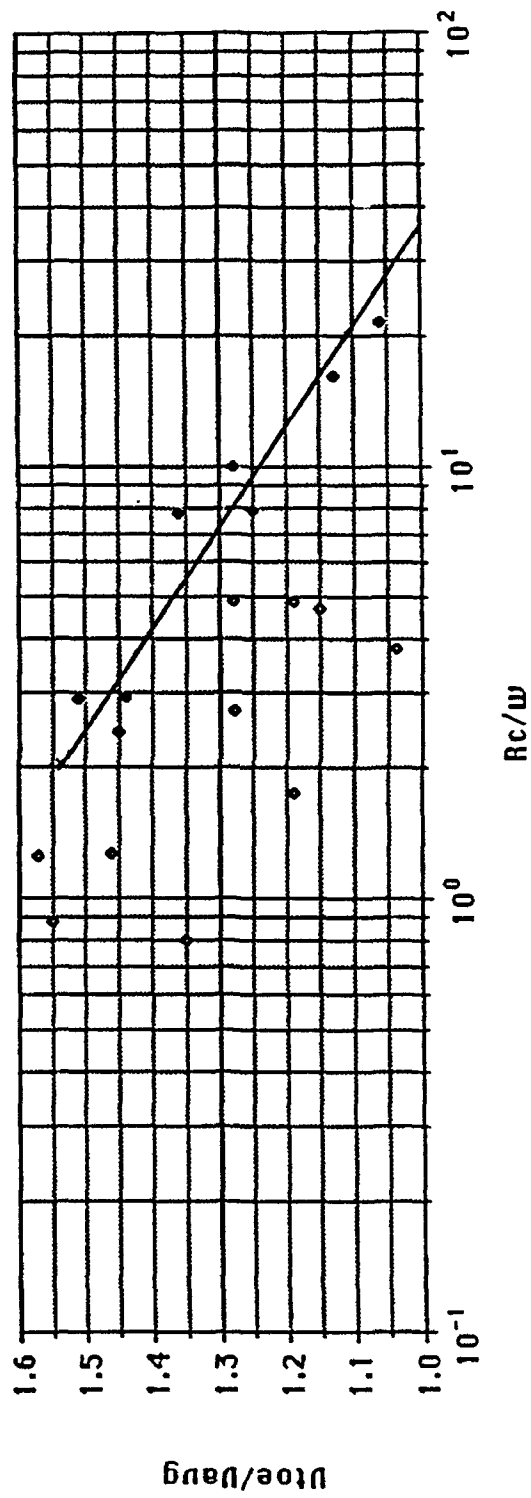
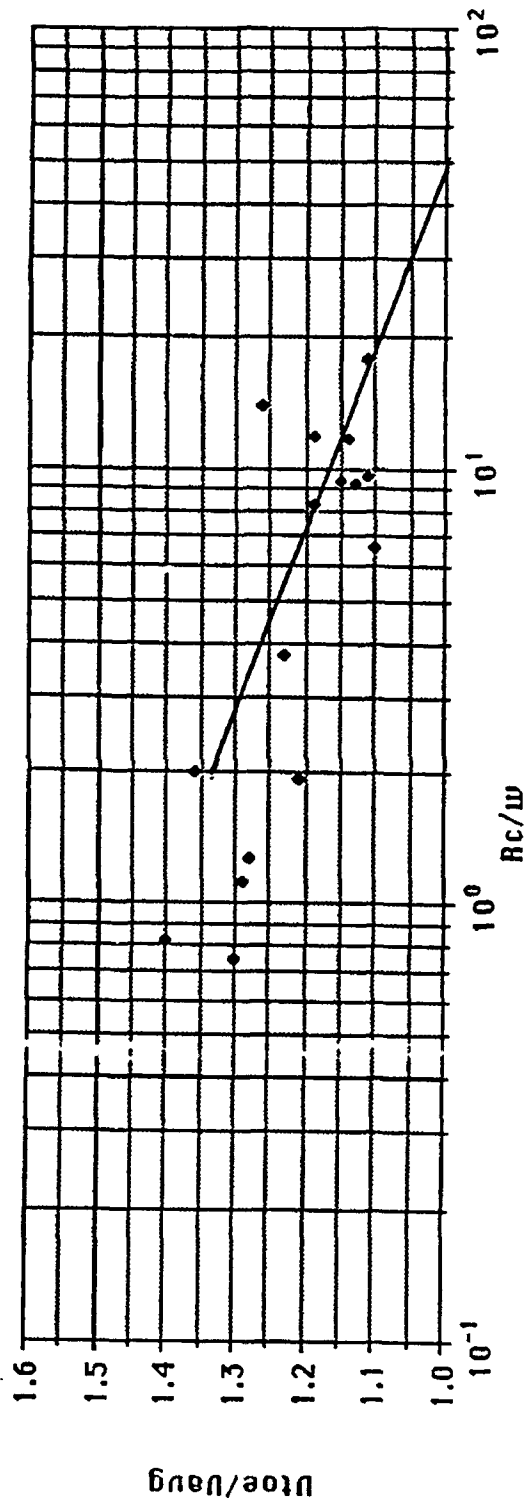


Fig. 20 Natural Rivers - Meandering Entrance



The adjusted coefficient of determination for this equation (r^2) is 0.87, again indicating good linearity at the upper edge of the data cloud. The line produced by this equation is shown in Fig. 20.

Trapezoidal and Rectangular Channels

There was no obvious difference between the data clouds for trapezoidal and rectangular channels, but the state of bed mobility did appear to affect the outer bank to average velocity ratio for laboratory channels. The plots for mobile and fixed bed channels are shown in Figs. 21 and 22 respectively. Hence, separate analyses were carried out for mobile and fixed-bed channels.

Laboratory Channels with Mobile Beds

The points used to define the upper boundary are indicated in Fig. 21. There was insufficient range in the other variables (L/w , w/d , $d/D50$, a) to allow their meaningful inclusion in the analysis. Hence a predictive equation based on Rc/w was produced. This is:

$$\frac{V_{TOE}}{V_{AVG}} = 1.55 - 0.41 \log \left(\frac{Rc}{w} \right) \quad (9)$$

The adjusted coefficient of determination for this equation (r^2) is 0.98, indicating good linearity at the upper edge of the data cloud. The line produced by this equation is shown in Fig. 21.

Laboratory Channels with Immobile Beds

The points used to define the upper boundary are indicated in Fig. 22. The WES diagram (Fig. 1), indicates that V_{toe}/V_{avg} is independent of Rc/w for Rc/w values greater than 6. Accordingly, only points with Rc/w values less than 6 were used to define the upper boundary. There was insufficient range in the other variables (L/w , w/d , $d/D50$, a) to allow their meaningful inclusion in the analysis. Hence a predictive equation based solely on Rc/w was produced. This is:

$$\frac{V_{TOE}}{V_{AVG}} = 1.73 - 0.89 \log \left(\frac{Rc}{w} \right) \quad (10)$$

The line produced by this equation is shown in Fig. 22.

Fig. 21 Trapezoidal & Rectangular Channels with mobile Beds

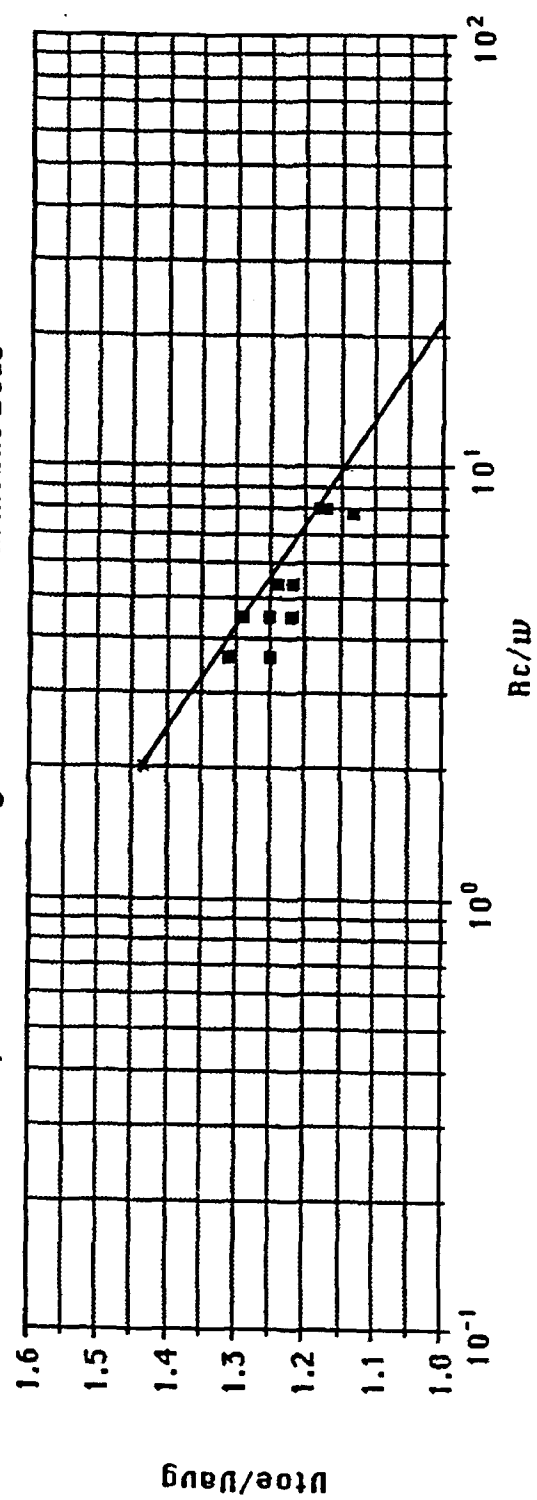
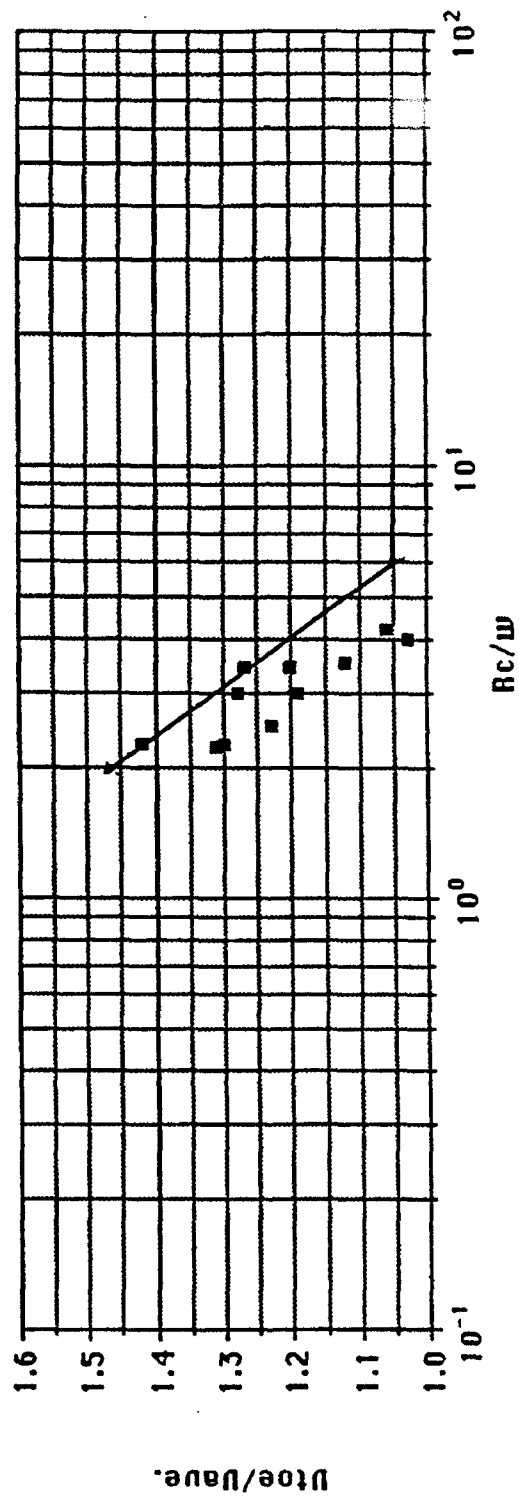


Fig. 22 Trapezoidal & Rectangular Channels with Immobile Beds



Prediction of Outer Bank Shear Stress in Open Channel Bendways

Very little reliable data on shear stress distributions in natural channels could be obtained. Consequently, the analysis was limited to data from laboratory flumes. It should be noted that in very few cases were both velocity and shear stress data available from the same series of experiments. Therefore, the data points used in this part of the study do not correspond to those used to investigate V_{toe}/V_{avg} .

Nevertheless, experience gained in the analysis of V_{toe}/V_{avg} did suggest that the state of bed mobility might affect the shear stress ratio for a bend. The plots for mobile and fixed bed channels are shown in Figs. 23 and 24 respectively. Separate analyses were performed for channels with mobile and immobile beds.

There was insufficient data to support multiple regression, and so analysis was limited to simple regression on upper boundary points to predict t_b/t_o as a function of Rc/w .

Laboratory Channels with Mobile Beds

The points used to fit a line to the data are indicated in Fig. 23. The equation of the resulting line is:

$$\frac{t_b}{t_o} = 3.31 \left(\frac{R}{w} \right)^{-0.5} \quad (11)$$

which is almost identical to the equation of the WES design curve for rough channels. However, this line is based on very limited data. As a result the top of the data cloud is poorly defined, producing an unacceptably low regression coefficient of only 0.27 for the upper boundary line. The line is shown in Fig. 23.

Further data from natural and mobile-bed laboratory channels are urgently needed to confirm the form, exponent and constant in this equation.

Laboratory Channels with Immobile Beds

The points used to fit a line to the data are indicated in Fig. 23. The WES diagram limits analysis to bends with Rc/w values between 1 and 5. For consistency, these limits were applied to the data used here also. The equation of the resulting line is:

$$\frac{t_b}{t_o} = 2.92 \left(\frac{R}{w} \right)^{-0.5} \quad (12)$$

which is very similar to the WES design equation for smooth channels. The high correlation coefficient $r = 0.96$ indicates good linearity in the upper boundary of the data.

Fig. 23 Rectangular Channels with Mobile Beds

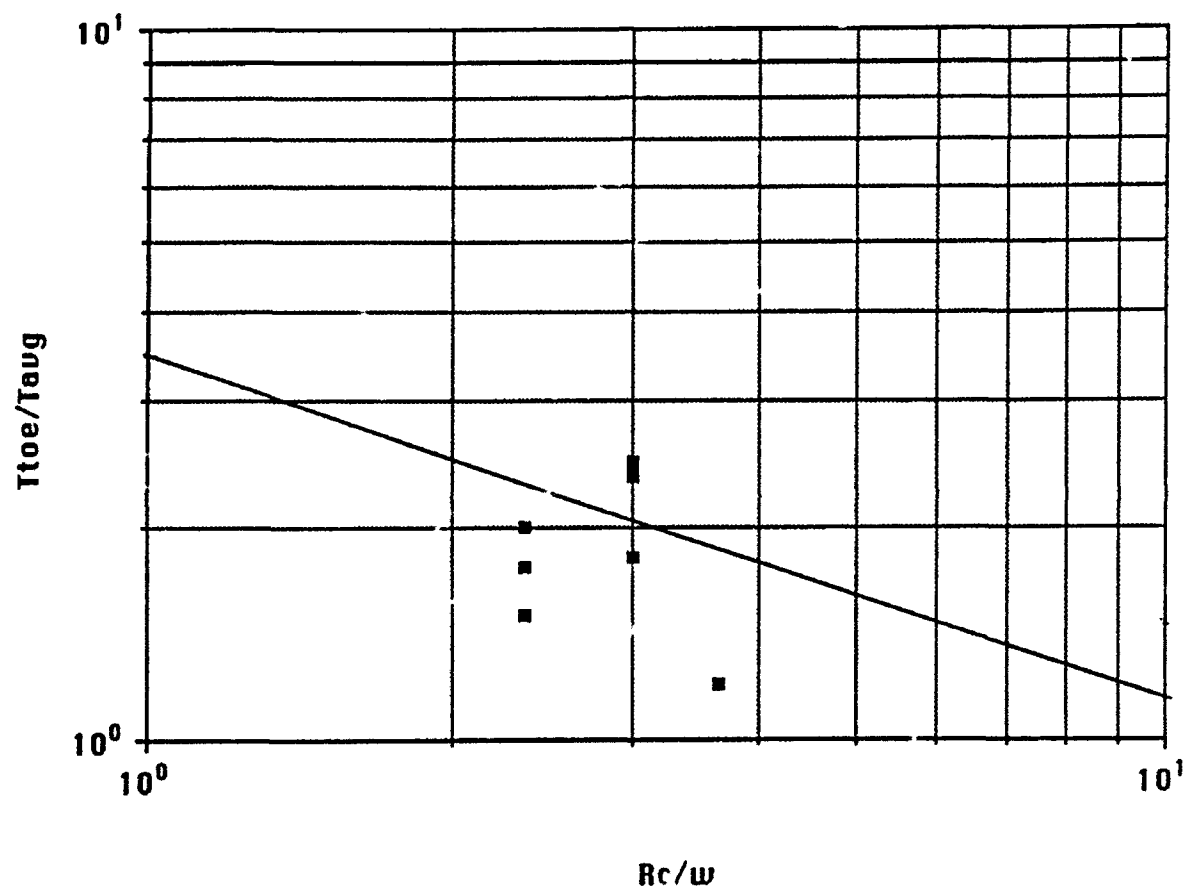
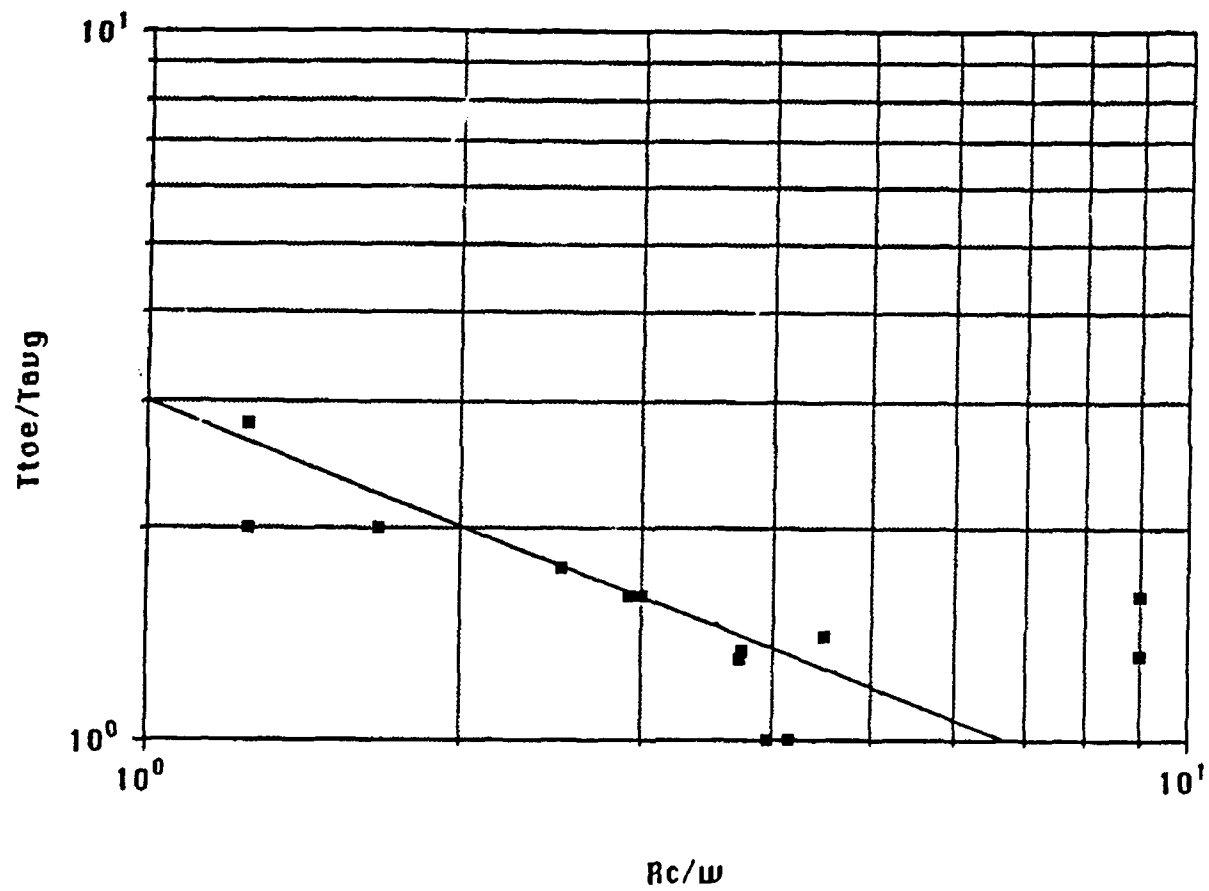


Fig. 24 Trapezoidal & Rectangular Channels with Immobile Beds



Modeling Approach

Background

During the last decade, a large number of papers presenting mathematical models for predicting the flow and bed topography in river bends has appeared in the literature. The authors have expressed a variety of opinions regarding the relative importance of the factors which control meander morphology. Most models are intended for use by sedimentologists interested in reconstructing past environments (eg. Allen, 1970, Bridge, 1976), or by river engineers attempting to predict the distribution of scour as an aid in the design of successful channel stabilization schemes (eg Odgaard, 1987). However, many models may be criticized because they are unnecessarily complicated and esoteric for these tasks, and because no attempt is made by the author(s) to recommend where, when, and how they should be applied to natural waterways. Some very pertinent remarks were made concerning the role of mathematical models in fluvial geomorphology and river engineering during a discussion by conference participants at the concluding session of the ASCE Rivers '83 conference, New Orleans, 1983 (Elliott, 1983). These are encapsulated by a comment from Charles Neill, a practising river engineer from Northwest Hydraulic Consultants, Canada. He said,

"It is important that mathematical models should have a good familiarity with the range of features encountered. ...It would be a service to the profession if these could be used to produce generalized tables, graphs or programs that would enable reasonable estimates of velocity and shear distributions to be made by practising engineers, without the necessity of access to an elaborate modelling facility" (Neill, 1983)

Difficulties are often encountered when attempting to apply models to natural systems. If the model requires certain input parameters that need to be known or to be measured in the field (the centreline mean velocity or the mean Darcy-Weisbach friction factor are examples), it is often difficult to assign a value with confidence. Often, field measurements unavailable and estimates are unreliable, and it may be the case that the model output may highly sensitive to incorrect values having been assigned to the input parameters.

Also, many models are written by, and apparently for, researchers. Unless you are a specialist, expert in programming, three dimensional fluid mechanics, and mathematics, it is virtually impossible to use them for a real world application without detailed assistance from the author of the computer coding.

In this part of the project, we examined a number of mathematical models, hoping to select several to try as predictors of outerbank velocity

in bendways. Before going on to report the results from the models selected, it is relevant to present a short outline of the physical basis for the models.

Basic Principles of Numerical Modeling of Bend Flow

The numerical modelling of flow and sediment processes in river bends is a subject that is receiving ever-increasing attention, and a large number of models are available. There are two components to most models. The first involves a solution to the equations of motion for fluid flow. The second is the interaction between the flow and the bed topography. This requires balancing the different forces acting on bed-material particles to produce an equilibrium bed topography.

All flow models start with the equations of motion for fluid flow. For application to bend flow, the equations are usually written in cylindrical coordinates. Given below are the equations of motion for the steady flow of an incompressible fluid in an orthogonal cartesian coordinate system (Rozovskii, 1957). The velocity components in the *s* (streamwise), *n* (perpendicular to *s*-axis), and *z* (vertical upwards from the stream bed) directions are denoted *u*, *v*, and *w* respectively, *r* = local radius of curvature; *p* = pressure; and *F* = friction term in the *s*, *n* and *z* directions respectively.

$$u \frac{\partial u}{\partial s} + v \frac{\partial u}{\partial n} + w \frac{\partial u}{\partial z} + \frac{uv}{r} = -\frac{1}{r} \frac{\partial p}{\partial s} + F_s,$$

$$u \frac{\partial v}{\partial s} + v \frac{\partial v}{\partial n} + w \frac{\partial v}{\partial z} - \frac{u^2}{r} = -\frac{1}{r} \frac{\partial p}{\partial n} + F_n$$

$$u \frac{\partial w}{\partial s} + v \frac{\partial w}{\partial n} + w \frac{\partial w}{\partial z} + g = -\frac{1}{r} \frac{\partial p}{\partial z} + F_z$$

The left hand sides of these equations are the convective acceleration terms. There are no local acceleration terms ($\partial/\partial t$) and so strictly speaking the models are not prognostic but diagnostic.

The continuity equation for 3-dimensional, incompressible flow is also specified:

$$\frac{\partial u}{\partial s} + \frac{1}{r} \frac{\partial(vr)}{\partial n} + \frac{\partial w}{\partial z} = 0$$

The principal cross-stream and downstream force balances are between:

- 1) the centrifugal and pressure gradient forces in the cross-stream direction; and
- 2) the downstream balance between gravitational and frictional forces.

Secondary circulation is usually considered to be the most important effect of curvature on flow, but the tilting of the water surface is also very important, because it alters the downstream slope of the water surface, generating large cross-stream variation in the downstream boundary shear stress and velocity fields.

Flow and bed topography models attempt to simulate the bed morphology of a channel bend by assuming that, at equilibrium, the forces directed inwards and outwards on each bed particle are balanced (Allen, 1970; Bridge, 1977). This means that particles of different sizes travel along paths of equal depth along the channel, under the influence of longstream drag. Models that use this scheme differ in the way in which the forces of lift and longstream drag are determined. The balance of forces acting on a particle in the mean flow direction is:

$$F_D \cos d = (W - F_L) \cos a \tan j$$

where F_D = drag force, d = deviation of the bed shear stress vector from the longstream direction, W = submerged weight, F_L = lift force, $\tan j$ is the dynamic friction coefficient due to collisions with the bed and other grains, and a = transverse slope of the point bar surface (Bridge, 1977) (see figure 25).

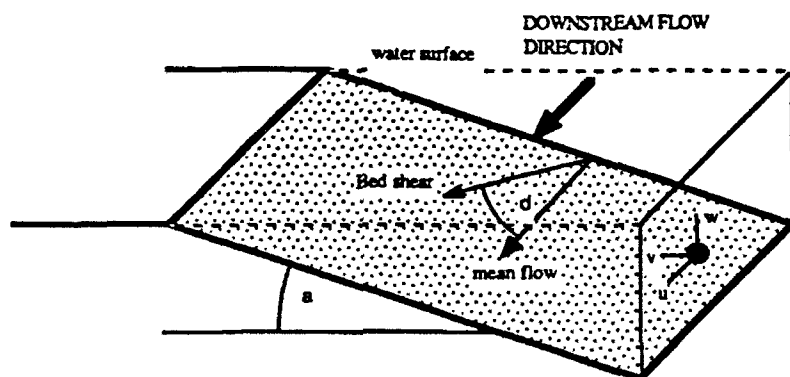


Figure 25 Definition diagram for flow in an open channel bend (adapted from Bridge, 1977)

The transverse force balance is therefore:

$$F_D \sin d = (W - F_L) \sin a$$

For any given point on the transverse bed profile, the balance of drag and immersed weight components acting on a bed particle is:

$$\tau_x \tan d = \frac{4}{3} p (D/2)^3 (s - r) g \sin \alpha$$

Where D = particle diameter, τ_x = longstream bed shear stress, and s and r = the sediment and fluid densities, respectively.

The theory assumes that the particles are moving as contact load. It is also important to note that for suspended particles, forces due to lift and the cross-stream component of particle weight are insignificant. Another assumption common to many models is that the angular deviation (d) of the shear stress vector from the downstream direction (and therefore the local transverse bed slope) is proportional to the ratio between the depth and radius of curvature:

$$\tan d = C \frac{h}{r}$$

where C is an empirical coefficient. This relation, developed by Rozovskii (1957) from the equations of motion, is actually applicable only to fully-developed secondary flow. Fully-developed flow occurs in the downstream part of long, constant-radius reaches where flow and bed topography remain constant with distance downstream and are independent of upstream conditions. For developing flow where flow and bed topography do not remain constant with distance, the governing equations are more complex and difficult to solve.

As the outward component of gravity is proportional to the cube of the diameter of the grain whilst the inward-acting drag on the particle is proportional to the square of the diameter. This leads to a sorting mechanism, recognized as an important process in meander bends by Allen (1970), Bridge (1977), Dietrich and Smith (1984) and Parker and Andrews (1985), whereby for the same velocity, larger particles will tend to roll, due to gravity, out towards the pool, while smaller ones will tend to be swept inwards by fluid drag (Fig. 26). Wilson (1973) used the same principle to propose an explanation for sorting in straight channels.

Review of Important Bendflow Models

Most researchers begin their analyses with the equations of motion and continuity, then simplify them until a solution is possible (eg. Dietrich and Smith, 1983; Odgaard, 1987). Others use simpler concepts of momentum and force, and treat the flow as a single unit, using depth-averaged equations (eg. Dietrich, 1988). The problem with the latter approach is that the effect of the secondary currents cannot be accounted for by the governing equations.

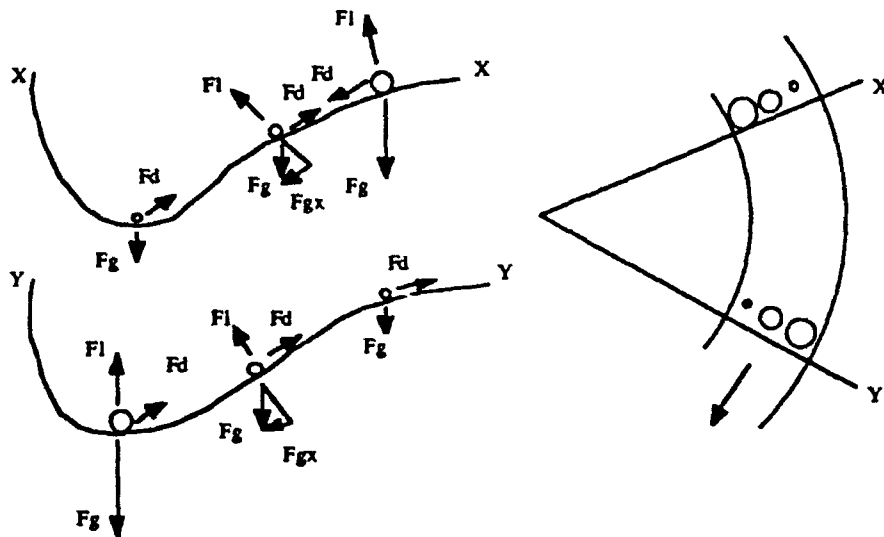


Figure 26 Forces acting on a particle at or near the bed. Opposing frictional resistance of the bed not shown. Forces are lift (F_l), drag (F_d), gravity (F_g), and the cross-stream component of gravity (F_{gx}). The gravitational force is proportional to the cube of the particle diameter, and the fluid forces are proportional to the square of the diameter. The result is a sorting mechanism. (Adapted from Dietrich, 1988)

Authors using the former method can control the secondary currents and the bed topography using these equations, but one needs to ask whether the simplifying assumptions made in order to solve the governing equations are justifiable for flow in bends.

The first important contribution was from a Dutch engineer, L Van Bendegom, in 1947, who formulated a model similar in principle to the fundamental model described in the previous section, and was the first to hypothesize that transverse bed slope bears a simple inverse relation to grain size. A complete appraisal of his work can be found in Allen (1978). Independent work by Rozovskii (1957) produced very similar results. Rozovskii developed a model for two-dimensional flow using the equations of motion. He later extended his analysis to consider flow in three dimensions and finally used his model to consider the development and decay of the helical flow cell. Much later, Engelund (1974) used an analysis of fluid motion developed by Rozovskii to produce a model for flow and bed topography. He claimed that the velocity defect law best describes velocity distribution in rivers. He first approximated two-dimensional bend flow, and subsequently included some second-order calculations to take into account the effect on the flow field of radial variations of depth and velocity. Bridge (1978) used Engelund's approach for bends approximating sine-generated paths. In his model, there is no cross-stream discharge of sediment at equilibrium, and grain size increases with increasing shear stress, so that the maximum sediment transport is towards the centre of the channel.

There has been much criticism of the approach taken by Engelund and Bridge. Dietrich and Smith (1984) point out that there are several deficiencies in the flow and the force balance equations. In particular, they argue that convective accelerations should not be considered second-order effects, but are important enough to be considered zero-order, ie. of primary importance. Although this hypothesis may be correct, Dietrich and Smith found that their model is very sensitive to the convective acceleration terms. This means that to apply their model successfully, measurements of the elevation of the water surface have to be accurate to within a fraction of a millimetre (Anthony, 1987). They claim their results to be supported by the flume data of Yen (1970). The implication of their argument is that the core of maximum sediment transport crosses the channel in a bend. Because of convective accelerations, the equilibrium bed slope is one that causes sufficient cross-stream bedload transport against the inward secondary currents, that the zone of increasing boundary shear stress in the pool is balanced by a convergence of sediment transport; the cross-stream sediment discharge is not zero. In addition, they describe a more sophisticated sorting procedure for sand-bed channels where the migrating shear stress field causes skewing of dune orientation. In the upstream part of the bend, where the maximum shear stress is near the inner bank, the inner bank end of dunes will migrate forward more rapidly than the outer bank ends, so the dunes will be skewed across the channel. Troughwise currents in the lee of the dune cause inward transport of fine particles while larger ones roll outwards. Near the bend exit the shear stress peak, having crossed the channel, causes the outer-bank end of the dunes to migrate more rapidly than the inner bank end, causing troughwise transport towards the pool even against the action of the main helix (figure 27). This is in addition to the simple force balance sorting scheme proposed by Wilson (1973) which still operates on the stoss side of the dunes. They argue that only where the shear stress field does not vary in the downstream direction (at the downstream end of a bend) will a particle force balance that assumes no net cross-stream transport apply.

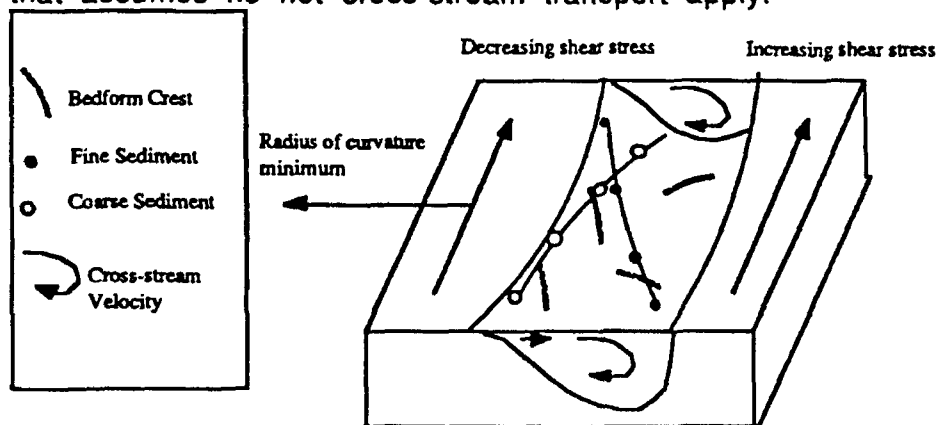


Figure 27 Sediment pathlines through a sand-bed meander bend. (Adapted from Dietrich and Smith, 1984a).

Smith and Mclean (1984) further developed Dietrich and Smith's (1983) model, and successfully tested it against the flume data of Hooke (1975).

Zimmerman and Kennedy (1978) proposed a model for transverse bed slope. They considered the spatially varying centrifugal force as exerting a torque force on the water. They balanced this force against the friction of the bed. They tested their results against flume data and agreement was generally good except in the outer bank region. However, their model is only applicable for fully-developed flow and can only produce a linear transverse bed slope, although linear transverse bed slopes are seldom found in river bends.

Another important distinction can be made between models that deal with fully-developed flow and models that deal with developing flow. Most models deal with fully-developed flow, but Odgaard (1986a & b) argued that an understanding of the role of developing flow is critical if erosion and deposition in river bends is to be understood fully. He proposed a model, based on a solution to the equations for conservation of mass, conservation of momentum, and lateral stability of the stream bed, that accounts for both developing flow, and convective accelerations. With flow conditions varying in the downstream direction, the governing equations are complex and difficult to solve, so models in this class need many simplifying assumptions. Kalkwijk and DeVriend (1980) wrote a model for two-dimensional flow in order to simulate flow in river bends where the depth is small compared to the width, and the width is small compared to the radius of curvature. They found good agreement between the results of their model and flume data. Discrepancies were found between the measured and theoretical velocity profiles at the outer bank, and at the entrance to the bend where the theoretical results were rather high. DeVriend and Geldof (1983) developed Kalkwijk and DeVriend's (1980) model and tested it using data from two bends on the River Dommel, Holland. The agreement between measured and computed results was generally good, but the model did not work well for the bend exits because they did not adequately account for the effect of secondary flow.

Chang(1984) developed a model (FLUVIAL-12) applicable to curved alluvial streams with non-erodible banks, and able to simulate stream bed changes during a given flow. FLUVIAL-12 is unique in that it can account for changes in stage. The model incorporates the major effects of helical flow, and performed well when tested with data from the San Lorenzo River.

Models Used

Two models were applied to predict the ratio of outer bank to average velocities in bendways: those of Bridge (1982) and Odgaard (1988). These models were unusual in that their authors were willing to make them fully available to us and assist us in their application. Most modellers do not

release their models in this way for a variety of reasons. Others supplied copies of papers reporting their models, but not computer codes. The task of rewriting entire codes for extremely complex models was simply beyond the resources available to this project.

The details of the models may be found in papers authored by Bridge (1982) and by Odgaard (1988). Some additional parameters were required for application of the models. These are listed in Appendix B.

Menu-driven FORTRAN programs were produced by a research associate, Dr Andrew Markham on the basis of complete codings supplied by the authors. A disk with the menu-driven programs may be found at the end of this report, in Appendix C. These programs were used to produce estimates of the maximum depth averaged velocity over the toe of the outer bank at each natural and laboratory channel bend.

Results of Model Applications

The results are listed in Table 11. Bridge's model failed for bends with very low radius to width ratios ($Rc/w < 1$), which caused the model to crash. Odgaard's model failed in many more cases. The problem was that in long bends the model predicted negative water depths at the inner bank, leading to its crashing. Further work on the model has so far failed to resolve this problem, although a solution must be possible. Probably, it will be essential to work directly with Prof. Odgaard to solve it.

Analysis of Results

The results are plotted as observed versus predicted outer bank velocities in Fig. 28. The agreement is generally quite good, although systematic errors are apparent in both graphs. In the case of Bridge's model these can be largely explained by a plot of the percent error, defined by:

$$\text{error} = \frac{(\text{observed } V_{\text{oe}} - \text{predicted } V_{\text{oe}})}{\text{observed } V_{\text{oe}}} \times 100 \% \quad (12)$$

versus the Rc/w ratio. The results are plotted in Fig 29. Bridge's model is seen to give generally excellent accuracy for bends with Rc/w values greater than 2, but to be unacceptable for tighter bends with $Rc/w < 2$. This accords with the theory of bend flow, the results of the data based approach and Bridge's own guidelines on the use of his model. More data are needed for Odgaard's model, but the preliminary results show that it is prone to underestimating the toe velocity by up to about 30 percent. This occurs because for bends with coarse sand or gravel beds the model does not predict the bed scouring at the outer bank which is observed in nature. Outer bank depths are only marginally greater than centerline depths, and likewise outer bank velocities are only slightly greater than the centerline velocity. However, the errors do not increase markedly at low Rc/w values, and the model is consistent right down to $Rc/w = 0.8$.

Table 11. Results of Model Tests

	Observed Vtoe	Rc/w	Bridge's Vtoe	Error (%)	Odgaard Vtoe	Error (%)
1	0.80	2.87	0.90	-12.500	0.52	35.000
2	0.80	0.82			0.56	30.000
3	0.60	0.75			0.42	30.000
4	1.10	0.88			0.67	39.100
5	1.35	1.75	1.70	-25.900		
6	0.80	1.25	0.45	43.800	0.57	28.700
7	0.74	1.27	1.25	-68.900	0.66	10.800
8	0.70	1.27	0.92	-31.400		
9	0.69	2.92	0.70	-1.400		
10	1.05	7.92	0.95	9.500		
11	0.95	6.58	0.99	-4.200		
12	1.03	3.76	1.02	0.971		
13	0.80	1.12	1.06	-32.500		
14	0.93	0.80				
15	0.83	7.83	0.72	13.300		
16	0.74	1.94	0.99	-33.800		
17	0.70	4.70	0.78	-11.400		
18	0.98	3.80	1.59	-62.200	0.93	5.100
19	1.60	4.84	1.39	13.100	1.35	15.600
20	1.46	17.90	1.48	-1.400		
21	1.53	16.23	1.48	3.300		
22	1.62	10.00	1.47	9.300		
23	1.55	9.34	1.56	-0.645		
24	1.69	8.23	1.70	-0.592		
25	1.81	13.85	1.62	10.500		
26	1.55	9.64	1.63	-5.200		
27	1.55	9.26	1.54	0.645		
28	1.67	11.75	1.66	0.599		
29	1.54	21.62	1.58	-2.600		
30	1.69	11.92	1.84	-8.900		
31	0.55	2.72	0.57	-3.600	0.45	18.200
32	0.61	2.42	0.50	18.000	0.45	26.200
33	0.75	2.00	0.67	10.700	0.60	20.000
34	1.60	4.85	1.77	-10.600		
35	0.52	4.5	0.50	3.800	0.40	23.100
36	0.55	4.5	0.56	-1.800	0.47	14.500
37	0.60	4.5	0.59	1.700	0.47	21.700
38	0.46	8.0	0.46	0.000	0.41	10.900
39	0.48	8.0	0.48	0.000	0.43	10.400
40	0.35	3.0	0.29	17.100	0.30	14.300
41	0.35	3.0	0.29	17.100	0.30	14.300
42	0.55		0.55	0.000	0.49	10.900
43	0.73		0.70	4.100	0.66	9.600
44	1.36		1.06	22.100	1.06	22.100
45	1.39		1.05	24.500	1.08	22.300
46	0.73		0.76	-4.100	0.57	21.900
47	0.68		0.76	-11.800	0.57	16.200
48	0.42		0.35	16.700	0.37	11.900

Fig. 28 Results of Model Tests

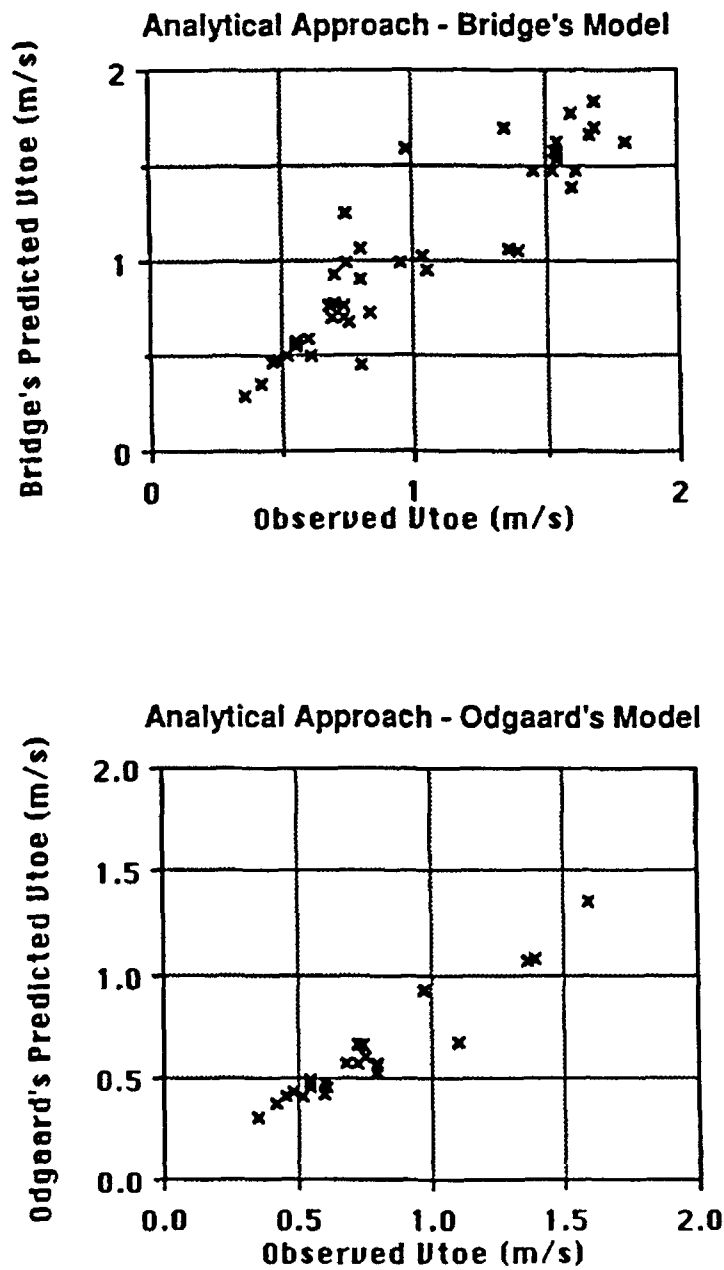
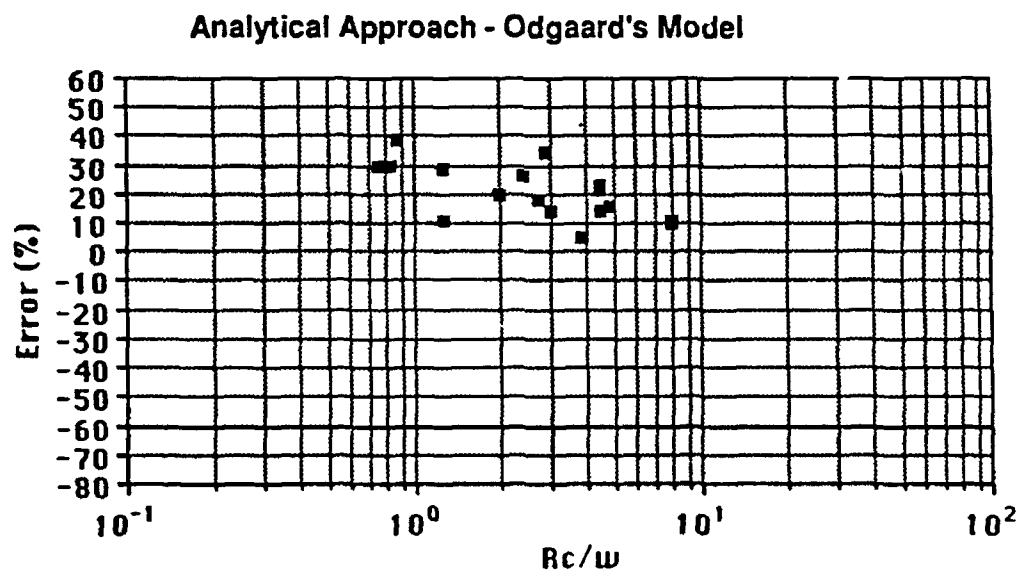
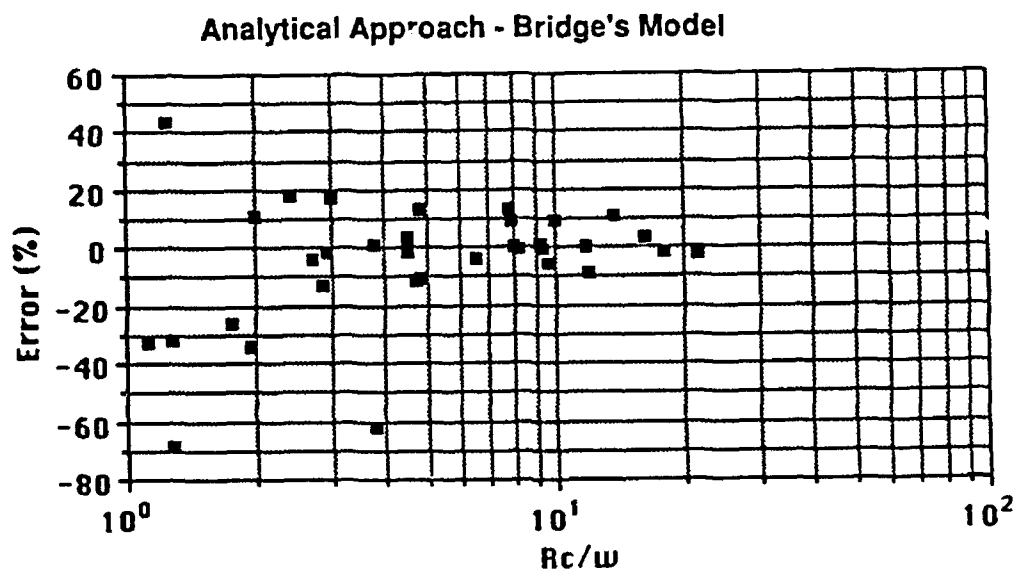


Fig. 29 Error Distributions as a Function of Rc/w



Prediction of Outer Bank Velocity in Natural Channel Bendways

On the basis of this test, Bridge's model can be relied upon to predict the outer bank velocity in a bend to within approximately ± 15 percent for bends with Rc/w greater than 2. This would appear to make it a strong candidate for adoption as a design method. For engineering design purposes it might be desirable to introduce a factor of safety of 1.15 to the predicted velocity to ensure that any error is on the safe side. Bridge's model should definitely not be used for bends with $Rc/w < 2$, where it is prone to large errors.

Bridge's model is available in menu driven format for IBM PC computers or compatible machines, and it is relatively easy to use (Appendix A).

Odgaard's model could not be fully tested as it crashed for many of the bends studied. For the bends at which it did work, errors were between 5 and 40 percent, for bends with Rc/w ratios between 0.8 and 5. Further work is needed before Odgaard's model could be recommended for use as a design method, but the fact that it does not fail for short, tight radius bends is very encouraging.

Conclusions

The prediction of outer bank velocity in bendways is a complicated problem. This velocity is affected by a number of factors, which are themselves closely inter-related. A data based approach seeks to develop a predictive equation by examining the relationship between independent and dependent variables illustrated by observations in real flows. An analytical approach attempts to use basic physics and the laws of motion to derive equations describing bend flow. It must be remembered though that even the "analytical" approach involves assumptions concerning the three dimensional nature of the flow, and calibration using empirical data.

To a scientist the analytical approach is preferable philosophically, provided that the underlying theory is sound. It is not yet agreed that this is so for the models tested here, and so they merit use only if they can produce acceptable results. This appears to be the case for the Bridge model.

Recommendations

It would be foolish to base a new design method on the results of a single, relatively small, study. Consequently, it is recommended that:

1. Follow-up work be undertaken to verify the findings reported here. In particular, further experimental work is needed in laboratory channels with a mobile bed and static banks to investigate how the pattern of bed scour affects the distribution of velocity adjacent to

the bank.

2. Field data on shear stress distributions on the outer bank are essential if that approach to riprap sizing is to be sustained.
3. A major initiative on flow in very tight bends of $Rc/W < 2$ is needed. It is clear that results from longer radius bends cannot be back extrapolated into this zone. The flow in such bends can impinge on the outer bank at a very acute angle. This links the problem to that of impinging flow in braided channels.
4. A better definition of outer bank velocity is needed. This study has shown that in many cases the maximum flow attack on the outer bank occurs on the bank face some distance above the toe, rather than at the toe itself. This is not accounted for in the present design approach.
5. For natural channels it appears that different equations may be appropriate for bends with straight and meandering approach channels. If this is true in general, it has important implications for river training and stabilization.
6. Several of the equations produced here deserve further testing and consideration as design methods.
7. On the basis of this study, the model developed by John Bridge of the State University of New York at Binghamton appears to predict outer bank velocity to within ± 15 percent for bends with $Rc/w > 2$. It is therefore recommended as a possible design approach to riprap sizing.

REFERENCES ON RIVER MEANDER FLOW

- Anthony, D.J. 1987. 'Stage-dependant channel adjustments in a meandering river, Fall River, Colorado', thesis presented to Colorado State University, Fort Collins, Colorado in partial fulfillment of the requirements for the degree of Master of Science, 180pp.
- Bagnold, R.A. 1960. 'Some aspects of the shape of river meanders', USGS Professional Paper, 282E.
- Bagnold, R.A. 1966. 'An approach to the sediment transport problem from general physics', USGS Professional Paper, 422-I.
- Bagnold, R.A. 1980. 'An empirical correlation of bed-load transport rates in flumes and natural rivers', Proc. Royal Society of London Series A, 372, 453-473.
- Barrage, A. and Dracos, Th. 1976. 'Turbulence measurements in rivers', Proceedings of the 2nd International IAHR symposium on stochastic hydraulics, 2-4 August, 1976.
- Bathurst, J.C. 1977. 'Resistance to flow in rivers with stony beds', thesis presented to the School of Environmental Science, University of East Anglia, Norwich in complete fulfillment of the requirements of the degree of Doctor of Philosophy, 401pp.
- Bathurst, J.C. 1979. 'Distribution of boundary shear stress in rivers', in D.D. Rhodes and G.P. Williams (eds), Adjustments of the Fluvial System, Proceedings of the 10th annual geomorphology symposium, New York, 21-22 September, 1979.
- Bathurst, J.C. Thorne, C.R. and Hey, R.D. 1977. 'Direct measurements of secondary currents in river bends', Nature, 269, 504-506.
- Bathurst, J.C. Thorne, C.R. and Hey, R.D. 1979. 'Secondary flow and shear stress at river bends', Journal of the Hydraulics Division, Proc. ASCE, 105 (HY10), 1277-1295.
- Bejan, A. 1982. 'Theoretical explanation for the incipient formation of meanders in straight rivers', Geophysical Research Letters, 9 (8), 831-834.
- Best, J.L. and Reid, J. 1984. 'Separation zone at open-channel junctions', Journal of Hydraulic Engineering, 110 (11), 1588-1594.
- Beven, K.J. Kirkby, M.J. Schofield, N. and Tagg, A.F. 1984. 'Testing a physically-based flood forecasting model (Topmodel) for three UK catchments', Journal of Hydrology, 69, 119-143.
- Bhowmik, N.G. 1979. 'Hydraulics of Flow in the Kaskasia River, Illinois', Report of Investigation 91, Illinois State Water survey/RI-91/79.
- Bhowmik, N.G. 1982. 'Shear stress distribution and secondary currents in straight open channels'. In R.D. Hey, J.C. Bathurst, and C.R. Thorne (eds), Gravel-Bed Rivers, Chichester: Wiley, 36-51.
- Blair, T.C. 1987. 'Sedimentary processes, vertical stratification sequences and geomorphology of the Roaring River Alluvial Fan, Rocky Mountain National Park, Colorado', Journal of Sedimentary Petrology, 57 (1),

- Bray, D.I. and Ho, J.C.T. 1985. 'Boundary shear stress distribution in 60-degree rectangular open-channel bends', Proc. Canadian Society for Civil Engineers Annual Conference, Saskatoon. 319-339
- Brice, J.C. 1974. 'Evolution of meander loops', Geological Society of America Bulletin, 85, 581-586.
- Brice, J.C. 1983. 'Planform properties of meandering rivers', in River Meandering, Proceedings of the conference Rivers '83, New Orleans, Louisiana, 24-26 October, 1983, 1-14.
- Bridge, J.S. 1975. 'Computer simulation of sedimentation in meandering streams', Sedimentology, 22, 3-43
- Bridge, J.S. 1976. 'Mathematical model and FORTRAN IV program to predict flow, bed topography and grain size in open-channel bends', Computers and Geosciences, 2, 407-416.
- Bridge, J.S. 1977. 'Flow, bed topography, grain size and sedimentary structure in open-channel bends: a three-dimensional model', Earth Surface Processes, 2, 401-416.
- Bridge, J.S. 1978. 'Flow, bed topography, grain size and sedimentary structure in open-channel bends: a three-dimensional model: a reply', Earth Surface Processes, 3, 423-424.
- Bridge, J.S. 1982. 'A revised mathematical model and FORTRAN IV program to predict flow, bed topography and grain size in open-channel bends', Computers and Geosciences, 8 (1), 91-95.
- Bridge, J.S. 1984. 'Flow and sedimentary processes in river bends: comparison of field observation and theory', in River Meandering, Proceedings of the conference Rivers '83, New Orleans, Louisiana, 24-26 October, 1983, 857-872.
- Bridge, J.S. and Dominic, D.F. 1984. 'Bed load grain velocities and sediment transport rates', Water Resources Research, 20 (4), 476-490.
- Bridge, J.S. and Jarvis, J. 1977. 'Velocity profiles and bed shear stress over various bed configurations in a river bend', Earth Surface Processes, 2, 281-294.
- Bridge, J.S. and Jarvis, J. 1982. 'The dynamics of a river bend: a study in flow and sedimentary processes', Sedimentology, 29, 499-541.
- Brundett, E. and Baines, W.D. 'The production and diffusion of vorticity in duct flow', Journal of Fluid Mechanics, 19 (3), 375-394.
- Callander, R.A. 1978. 'River meandering', Annual Review of Fluid Mechanics, 10, 129-158.
- Carey, W.C. 1969. 'Formation of flood plain lands', Journal of the Hydraulics Division, Proc. ASCE, 95 (HY3), 981-994
- Carson, M.A. and Lapointe, M.F. 1983. 'The inherent asymmetry of river meander planform', Journal of Geology, 91, 41-55.
- Chalkin, T.M. and Francis, J.R.D. 1952. 'Discussion of: On the origin of river meanders, by P.W. Werrner', Transactions of the American Geophysical Union, 33 (5), 771-773.
- Chang, H.H. 1980. 'Stable alluvial canal design', Journal of the Hydraulics

- Division, Proc. ASCE, 106 (HY5), 873-891.
- Chang, H.H. 1984. 'Regular meander path model', *Journal of Hydraulic Engineering*, 110 (10), 1398-1411.
- Chang, H.H. 1985a. 'Water and sediment routing through curved channels', *Journal of Hydraulic Engineering*, 111 (4), 644-658.
- Chang, H.H. 1985b. 'Formation of alternate bars', *Journal of Hydraulic Engineering*, 111 (11), 1412-1420.
- Chang, H.H. Simons, D.B. and Woolhiser, D.A. 1971. 'Flume experiments on alternate bar formation', *Journal of the Waterways, Harbors and Coastal Engineering Division, Proc. ASCE*, 97 (WW1), 155-165.
- Chen, G. and Shen, H.W. 1983. 'River curvature-width ratio effect on shear stress', in *River Meandering, Proceedings of the conference Rivers '83*, New Orleans, Louisiana, 24-26 October, 1983, 687-699.
- Chitale, S.V. 1970. 'River channel patterns', *Journal of the Hydraulics Division, Proc. ASCE*, 96 (HY1), 201-221.
- Coleman, N.L. 1986. 'Effects of suspended sediment on the open-channel velocity distribution', *Water Resources Research*, 22 (10), 1377-1384.
- DeVriend, H.J. and Geldof, H.J. 1983. 'Main flow velocity in short river bends', *Journal of Hydraulic Engineering*, 109 (7), 991-1011.
- Dietrich, W.E. 1987. 'Flow and sediment transport in river bends', in K.S. Richards (ed), *River channels: environment and process*, Oxford: Blackwell, 391pp.
- Dietrich, W.E. and Smith, J.D. 1983. 'Influence of the point bar on flow through curved channels', *Water Resources Research*, 19 (5), 1173-1192.
- Dietrich, W.E. and Smith, J.D. 1984a. 'Bed load transport in a river meander', *Water Resources Research*, 20 (10), 1355-1380.
- Dietrich, W.E. and Smith, J.D. 1984b. 'Processes controlling the equilibrium bed morphology in river meanders', in *River Meandering, Proceedings of the conference Rivers '83*, New Orleans, Louisiana, 24-26 October, 1983, 759-769.
- Dietrich, W.E. Smith, J.D. and Dunne, T. 1979. 'Flow and sediment transport in a sand-bedded meander', *Journal of Geology*, 87, 305-315.
- Dietrich, W.E. Smith, J.D. and Dunne, T. 1984. 'Boundary shear stress, sediment transport and bed morphology in a sand-bedded river meander during low and high flow', in *River Meandering, Proceedings of the conference Rivers '83*, New Orleans, Louisiana, 24-26 October, 1983, 632-639.
- Draper, N.R. and Smith, H. 1966. *Applied regression analysis*, Chichester: Wiley, 407pp.
- Eakin, H.M. 1935. 'Diversity of current direction and load distribution on stream bends', *Transactions of the American Geophysical Union*, II, 467-472.
- Einstein, H.A. and Harder, J.A. 1954. 'Velocity distribution and the boundary layer at channel bends', *Transactions of the American Geophysical Union*, 35 (1), 114-120.

- Einstein, H.A. and Li, H. 1958. 'Secondary currents in straight channels', Transactions of the American Geophysical Union, 39 (6), 1085-1088.
- Einstein, H.A. and Shen, H.W. 1964. 'A study of meandering in straight alluvial channels', Journal of Geophysical Research, 69 (24), 5239-5247.
- Engelund, F. 1974. 'Flow and bed topography in channel bends', Journal of the Hydraulics Division, Proc. ASCE, 100 (HY11), 1631-1648.
- Ferguson, R.I. 1973. 'Regular meander path models', Water Resources Research, 9 (4), 1079-1086.
- Ferguson, R.I. 1977. 'Meander migration: equilibrium and change', in K.J. Gregory (ed), River Channel Changes, Chichester: Wiley. 235-248.
- Francis, J.R.D. and Asfari, A.F. 1970. 'Visualisation of spiral motion in curved open channel of large width', Nature, 225, 725-728.
- Friedkin, J.F. 1945. 'A laboratory study of the meandering of alluvial rivers', US Army Waterways Experiment Station, Vicksburg, Mississippi.
- Fujita, Y. and Muramoto, M. 1985. 'Studies on the process of development of alternate bars', Bulletin of the Disaster Research Institute, Kyoto University, 35 (314), 55-86.
- Gardner, R.H. Huff, D.D. O'Neill, R.V. Mankin, J.B. Carney, J. and Jones, J. 1980. 'Application of error analysis to a marsh hydrology model', Water Resources Research, 16 (4), 659-644.
- Gessner, F.B. 1972. 'The origin of secondary flow in turbulent flow along a corner', Journal of Fluid Mechanics, 58 (1), 1-25.
- Gessner, F.B. and Jones, J.B. 1965. 'On some aspects of fully developed turbulent flow in rectangular channels', Journal of Fluid Mechanics, 23 (4), 689-713.
- Gibson, A.H. 1908. 'On the depression of the filament of maximum velocity in a stream flowing through an open channel', Proc. Royal Society of London, Series A, 82, 149-159.
- Gulliver, J.S. and Halverson, M.J. 1987. 'Measurements of large streamwise vortices in an open channel flow', Water Resources Research, 23 (1), 115-123.
- Hack, J.T. and Young, R.S. 1959. 'Intrenched meanders of the north fork of the Shenandoah River, Virginia', USGS Professional paper, 354-A.
- Hawton, D.A. 1980. 'Problems and practicalities of the electromagnetic flowmeter in stream flow measurements', final year undergraduate project submitted to the School of Environmental Science, University of East Anglia, Norwich in partial fulfilment of the degree of Bachelor of Science, 63pp.
- Henderson, F.M. 1966. 'Open Channel Flow', London: Macmillan, 502pp.
- Hey, R.D. 1976. 'Geometry of river meanders', Nature, 262, p482.
- Hey, R.D. 1978. 'Determinate hydraulic geometry of river channels', Journal of the Hydraulics Division, Proc. ASCE, 104 (HY6), 869-885.
- Hey, R.D. 1979. 'Causal and functional relations in fluvial geomorphology', Earth Surface Processes, 4, 179-182.

- Hey, R.D. and Thorne, C.R. 1975. 'Secondary flow in river channels', *Area*, 7 (3), 191-195
- Hey, R.D. and Thorne, C.R. 1981. 'Flow processes and river channel morphology', in D.E. Walling, and T.P. Burt (eds.), *Catchment Experiments in Fluvial Geomorphology*, Norwich: Geo-Books.
- Hey, R.D. and Thorne, C.R. 1986. 'Stable channels with mobile gravel beds', *Journal of Hydraulic Engineering*, 12 (8), 671-689.
- Hickin, E.J. 1974. 'The development of meanders in natural river channels', *American Journal of Science*, 274, 414-442.
- Hickin, E.J. 1977. 'Hydraulic factors controlling channel migration', in R. Davidson-Arnott, and W. Nickling, (eds.), *Research in Fluvial Systems, Proceedings of the 5th Guelph Geomorphology Symposium*. Norwich: Geo-books, 59-72.
- Hickin, E.J. 1978. 'Mean flow structure in meanders of the Squamish River, British Columbia', *Canadian Journal of Earth Sciences*, 15 (11), 1833-1849.
- Hickin, E.J. and Nansen, G.C. 1975. 'The character of channel migration on the Beatton River, Northeast British Columbia, Canada', *Geological Society of America Bulletin*, 86, 487-494
- Hickin, E.J. and Nansen, G.C. 1984. 'Lateral migration rates of river bends', *Journal of Hydraulic Engineering*, 110 (11), 1557-1567.
- Hinze, J.O. 1967. 'Secondary currents in wall turbulence', *Physics of Fluids Supplement*, 10, 5122-5125.
- Hooke, J.M. 1987. 'Discussion of: Lateral migration rates of river bends, by E.J. Hickin and G.C. Nansen', *Journal of Hydraulic Engineering*, 113 (7), 915-918.
- Hooke, J.M. and Harvey, A.M. 1983. 'Meander changes in relation to bend morphology and secondary flows', in J.D. Collinson and J. Lewin (eds.), *Modern and Ancient Fluvial Systems*, IAS publication no. 6, Oxford, UK: Blackwell, 121-132.
- Hooke, R. leB. 1975. 'Distribution of sediment transport and shear stress in a meander bend', *Journal of Geology*, 83 (5), 543-565.
- Hooke, R.leB. and Chase, C.G. 1978. 'Discussion of: Flow, bed topography, grain size and sedimentary structure in open channel bends: a three-dimensional model, by J.S. Bridge', *Earth Surface Processes*, 3, 421-422.
- Ikeda, S. Parker, G. and Sawai, K. 1981. 'Bend theory of river meanders. Part I. Linear development', *Journal of Fluid Mechanics*, 112, 363-377.
- Ikeda, S. and Nishimura, T. 1985. 'Bed topography in bends of sand-silt rivers', *Journal of Hydraulic Engineering*, 111 (11), 1397-1411.
- Ippen, A.T. and Drinker, P.A. 1962. 'Boundary shear stress in curved trapezoidal channels', *Journal of the Hydraulics Division, Proc. ASCE*, 88 (HY5), 143-179.
- Jackson, R.G. 1975. 'Velocity-bedform-texture patterns of meander bends in the Lower Wabash River of Illinois and Indiana', *Geological Society*

- of America Bulletin, 86, 1511-1522.
- Jaeggi, M.N.R. 1987. 'Interaction of bed load transport with bars', in: R.D. Hey, J.C. Bathurst, and C.R. Thorne, (eds), 'Gravel Bed Rivers', Chichester: Wiley, 829-841.
- Julien, P.W. and Thorne, C.R. 'Discussion of: Flow and bed topography in alluvial channel bends, by A.J. Odgaard', Journal of Hydraulic Engineering, 111(6), 1031-1033.
- Kalkwijk, J.P.Th. and DeVriend, H.J. 1980. 'Computation of the flow in shallow river bends', Journal of Hydraulic Research, 18 (4), 327-342.
- Keller, E.A. 1972. 'Development of alluvial stream channels: a five-stage model', Geological Society of America Bulletin, 83 (2), 1531-1536.
- Kellerhals, R, Church, M, and Bray, D.I. 1976. 'Classification and analysis of river processes', Journal of the Hydraulics Division, Proc. ASCE, 102 (HY7), 813-829.
- Kirkby, M.J. Naden, P.S. Burt, T.P. and Butcher, D.P. 1987, 'Computer Simulation in Physical Geography', Chichester: Wiley, 227pp.
- Knighton, D. 1984. 'Fluvial Forms and Processes', London: Edward Arnold, 218pp.
- Langbein, W.B. and Leopold, L.B. 1966. 'River meanders-theory of minimum variance', USGS Professional Paper, 422-H.
- Lapointe, M.F. and Carson, M.A. 1986. 'Migration patterns of an assymetric meandering river: the Rogue River, Quebec', Water Resources Research, 22 (5), 731-743.
- Leeder, M.R. and Bridges, P.H. 1975. 'Flow separation in meander bends', Nature, 253, 338-339.
- Leliavsky, S. 1955. 'An Introduction to Fluvial Hydraulics', London: Constable and Co. 257pp.
- Leopold, L.B. 1982. 'Water surface configuration in river channels and implications for meander development', in: R.D. Hey, J.C. Bathurst, and C.R. Thorne, (eds), 'Gravel Bed Rivers', Chichester: Wiley.
- Leopold, L.B. Bagnold, R.A. Wolman, M.G. and Brush, L.M. 1960. 'Flow resistance in sinuous or irregular channels', USGS Professional Paper, 282-D.
- Leopold, L.B. and Langbein, W.B. 1966. 'River meanders', Scientific American, 214, 60-70.
- Leopold, L.B. and Wolman, M.G. 1957. 'River channel patterns-braided, meandering and straight', USGS Professional Paper, 282-B.
- Leopold, L.B. and Wolman, M.G. 1960. 'River meanders', Geological Society of America Bulletin, 71, 769-794.
- Leopold, L.B. Wolman, M.G. and Miller, J.P. 1964. 'Fluvial Processes in Geomorphology', San Francisco: Freeman.
- Lewin, J. 1976. 'Initiation of bed forms and meanders in coarse grained sediment', Geological Society of America Bulletin, 87, 281-285.
- Lewin, J. 1977. 'Channel pattern changes', in K.J. Gregory (ed.), River Channel Changes, Chichester: Wiley. 167-184.
- Lewin, J. and Brindle, B.J. 1977. 'Confined meanders', in K.J. Gregory (ed.),

- River Channel Changes, Chichester: Wiley. 221-233.
- Matthes, G.H. 1941. 'Basic aspects of stream meanders', Transactions of the American Geophysical Union, 22 (3), 632-636.
- McCrea, G.H. and Bray, D.I. 1984. 'Effects of cross-sectional shape on open channel bend flow', Proc. Canadian Society of Civil Engineers Annual Conference, Halifax, 515-529.
- McCutcheon, 1988. 'Discussion of: Meander flow model. I: Development', by A.J. Odgaard', Journal of Hydraulic Engineering, 819-822
- Muller, A. 1976. 'Effect of secondary flow on turbulence in an open channel flow', Proc. 2nd International IAHR conference on stochastic hydraulics, 2-4 August, 1976, 3.1-3.22
- Nansen, G.C. and Page, K. 1983. 'Lateral accretion of fine-grained concave benches on meandering rivers', in J.D. Collinson and J. Lewin (eds.), Modern and Ancient Fluvial Systems, IAS publication no. 6, Oxford, UK: Blackwell, 133-143.
- Nelson, J.M. 1988. 'Mechanics of flow and sediment transport over non-uniform erodible beds', thesis presented to the University of Washington, in partial fulfillment of the requirements of degree of Doctor of Philosophy, 228pp.
- Noble, C.A. and Palmquist, R.C. 1968. 'Meander growth in artificially straightened streams', Proc. Iowa Academy of Science, 75, 234-242.
- Odgaard, A.J. 1981. 'Transverse bed slope in alluvial channel bends', Journal of the Hydraulics Division, Proc. ASCE, 107 (HY12), 1677-1693.
- Odgaard, A.J. 1986. 'Meander flow model. I: Development', Journal of Hydraulic Engineering, 112 (12), 1117-1136.
- Odgaard, A.J. 1986. 'Meander flow model. II: Applications', Journal of Hydraulic Engineering, 112 (12), 1137-1150
- Odgaard, A.J. 1987. 'Streambank erosion along two rivers in Iowa', Water Resources Research, 23 (7), 1225-1236.
- Odgaard, A.J. 1989. 'River-meander model. I: Development', Journal of Hydraulic Engineering, Vol. 115, No. 11, 1433-1450.
- Odgaard, A.J. 1988. 'River-meander model. II: Applications', Journal of Hydraulic Engineering, Vol. 115, No. 11, 1450-1464.
- Odgaard, A.J. and Bergs, M.A. 1988. 'Flow processes in curved alluvial channels', Water Resources Research, 24 (1), 45-56.
- Osman, A.M. and Thorne, C.R. 1988. 'Riverbank stability analysis. I: Theory', Journal of Hydraulic Engineering, 114 (2), 134-150.
- Osterkamp, W.R. 1978. 'Gradient, discharge and particle-size relations of alluvial channels in Kansas, with observations on braiding', American Journal of Science, 278, 1253-1268.
- Parker, G. and Andrews, E.D. 1985. 'Sorting of bed load sediment by flow in meander bends', Water Resources Research, 21 (9), 1361-1373.
- Parker, G. Diplas, P. and Akiyama, J. 1983. 'Meander bends of high amplitude', Journal of Hydraulic Engineering, 109 (10), 1323-1337.
- Perkins, H.J. 1970. 'The formation of streamwise vorticity in turbulent

- flow', *Journal of Fluid Mechanics*, 44 (4), 721-740.
- Pitlick, J. and Harvey, M.D. 1987. 'Geomorphic response of Fall River following the Lawn Lake flood, Rocky Mountain National Park, Colorado', Final report, contract no. DAAG29-85-K-0108, US Army Research Office.
- Pitlick, J. and Thorne, C.R. 1987. 'Sediment supply, movement and storage in an unstable gravel-bed river', in C.R. Thorne, J.C. Bathurst, and R.D. Hey (eds.), *Sediment Transport in Gravel Bed Rivers*, Chichester: Wiley, 151-178.
- Prandtl, L. 1952. 'Essentials of Fluid Dynamics', London: Blackie, 452pp.
- Prus-Chacinski, T. 1954. 'Patterns of motion in open channel bends', *International Association of Scientific Hydrology*, 38 (3), 311-318.
- Quraishy, M.S. 1944. 'The origin of curves in rivers', *Current Science*, 2, 36-39.
- Quick, M.C. 1974. 'Mechanism for streamflow meandering', *Journal of the Hydraulics Division, Proc. ASCE*, 100 (HY6), 741-753.
- Rais, S. 1985. 'Analysis of flow close to the outer bank of a meander bend', thesis presented to Colorado State University, Fort Collins, Colorado, in partial fulfilment of the requirements for the degree of Doctor of Philosophy, 287pp.
- Raisz, D. 1955. 'Which way does a river meander?', *Photogrammetric Engineering*, 21, p738.
- Reid, J.B.Jr. 1983. 'Artificially induced concave bench deposition as a means of floodplain erosion control', in *River Meandering, Proceedings of the conference Rivers '83*, New Orleans, Louisiana, 24-26 October, 1983, 295-304.
- Richards, K.S. 1976. 'Morphology of riffle-pool sequences', *Earth Surface Processes*, 1, 71-88.
- Rozovskii, L.L. 1957. 'Flow of water in bends of open channels', translated by the Israel Programme for Scientific Translation (1961), 233pp.
- Schumm, S.A. 1963. 'Sinuosity of alluvial rivers on the Great Plains', *Geological Society of America Bulletin*, 74, 1089-1100.
- Schumm, S.A. 1977. 'The Fluvial System', New York: Wiley-Interscience, 338pp.
- Schumm, S.A. and Lichty, R.W. 1965. 'Time, space, and causality in geomorphology', *American Journal of Science*, 263, 110-119.
- Schumm, S.A. and Khan, H.R. 1972. 'Experimental study of channel patterns', *Geological Society of America Bulletin*, 83, 1755-1770.
- Shen, H.W. and Komura, S. 1968. 'Meandering tendencies in straight alluvial channels', *Journal of the Hydraulics Division, Proc. ASCE*, 94 (HY4), 997-1015.
- Siegenthaler, M.C. and Shen, H.W. 1983. 'Shear stress uncertainties in bends from equations', in *River Meandering, Proceedings of the conference Rivers '83*, New Orleans, Louisiana, 24-26 October, 1983, 662-674.
- Stearns, P.F. 1883. 'On the current meter, together with the reason why the maximum velocity of water flowing in open channels is below the

- surface', *Transactions of the American Society of Civil Engineers*, 12, 331-338.
- Swan, D. Clague, J.J. and Luternauer, J.L. 1979. 'Grain-size statistics 2: Evaluation of grouped moment measures', *Journal of Sedimentary Petrology*, 49 (2), 487-500.
- Tanner, F.W. 1960. 'Helicoidal flow, a possible cause of meandering', *Journal of Geophysical Research*, 65 (3), 993-995.
- Taylor, G. Crook, K.A.W. and Woodyer, K.D. 1971. 'Upstream-dipping foreset cross-stratification: Origin and implications for palaeoslope analysis', *Journal of Sedimentary Petrology*, 41 (1), 578-581.
- Thompson, A. 1986. 'Secondary flow and the pool-riffle unit: A case study of the processes of meander development', *Earth Surface Processes and Landforms*, 11, 631-641.
- Thomson, J. 1876. 'On the origins and winding of rivers in alluvial plains', *Proceedings of the Royal Society of London*, 25 (5), 5-8.
- Thorne, C.R. 1978. 'Processes of bank erosion in river channels', thesis submitted to the School of Environmental Science, University of East Anglia, Norwich, in complete fulfilment of the requirements of the degree of Doctor of Philosophy, 447pp.
- Thorne, C.R. and Lewin, J. 1979. 'Bank processes, bed material movement and planform development in a meandering river', in: D.D. Rhodes and G.P. Williams (eds), *Adjustments of the Fluvial System*, Iowa: Kendall/Hunt, 117-137.
- Thorne, C.R. and Osman, A.M. 1988. 'Riverbank stability analysis, 2: Applications', *Journal of Hydraulic Engineering*, 114 (2), 151-172.
- Thorne, C.R. Rais, S. Zevenbergen, L.W. Bradley, J.B. and Julien, P.Y. 1983. 'Measurements of bend flow hydraulics on the Fall River at low stage', *Water Resources Field Support Laboratory project report no. 83-9P*, Fort Collins, Colorado.
- Thorne, C.R. Zevenbergen, L.W. Bradley, J.B. and Pitlick, J. 1985. 'Measurements of bend flow hydraulics on the Fall River at bankfull stage', *Water Resources Division project report no. 85-3*, Fort Collins, Colorado.
- Thorne, C.R. Zevenbergen, L.W. Rais, S. Bradley, J.B. and Julien, P.Y. 1985. 'Direct measurements of secondary currents in a meandering sand-bed river', *Nature*, 315, 746-747.
- Thorne, C.R. 1982. 'Processes and mechanisms of bank erosion', in: R.D. Hey, J.C. Bathurst, and C.R. Thorne, (eds), *'Gravel Bed Rivers'*, Chichester: Wiley, 227-259.
- Thorne, C.R. and Hey, R.D. 1979. 'Direct measurements of secondary currents at a river inflection point', *Nature*, 280, 226-228.
- Thorne, C.R. and Rais, S. 1983. 'Secondary current measurements in a meandering river', in *River Meandering*, *Proceedings of the conference Rivers '83*, New Orleans, Louisiana, 24-26 October, 1983, 675-686.
- Van Alphen, J.S.L.J. Bloks, P.M. and Hoekstra, P. 1984. 'Flow and grain size pattern in a sharply curved river bend', *Earth Surface Processes and Landforms*, 9, 513-522.

- Varshney, D.V. and Garde, R.J. 1975. 'Shear distribution in bends in rectangular channels', *Journal of the Hydraulics Division, Proc. ASCE*, 101 (HY8), 1053-1066.
- Werner, P.W. 1951. 'On the origin of river meanders', *Transactions of the American Geophysical Union*, 32 (6), 898-902.
- Wilson, I.G. 1973. 'Equilibrium cross-section of meandering and braided rivers', *Nature*, 241, 393-394.
- Woodyer, K.D. 1970. Discussion of: 'Formation of flood plain lands', by W.C. Carey, *Journal of the Hydraulics Division, Proc. ASCE*, 96 (HY3), 849-850.
- Yalin, M.S. 1972. 'The Mechanics of Sediment Transport', Oxford: Pergamon, 298pp.
- Yang, C.T. 1971. 'On river meanders', *Journal of Hydrology*, 13, 231-253.
- Yen, C.L. 1970. 'Bed topography effect on flow in a meander', *Journal of the Hydraulics Division, Proc. ASCE*, 96 (HY1), 57-73.
- Yen, C.L. and Yen, B.C. 1971. 'Water surface configuration in channel bends', *Journal of the Hydraulics Division, Proc. ASCE*, 97 (HY2), 303-321.
- Zimmerman, C. and Kennedy, J.F. 1978. 'Transverse bed slope in curved alluvial streams', *Journal of the Hydraulics Division, Proc. ASCE*, 104 (HY1), 33-48.

APPENDIX A - SOURCES OF DATA USED IN THIS PROJECT

Source of data for natural channels.

Anthony, D.J. (1987). "Stage dependent channel adjustments in a meandering river, Fall River, Colorado." MSc. Thesis. Colorado State Univ.

Bathurst, J.C. (1977). "Resistance to flow in rivers with stoney beds." PhD. Thesis. Univ. of East Anglia. U.K.

Bhowmik, N.G. (1979). "Hydraulics of flow in the Kaskaskia River, Illinois." Report of Investigation 91. ISWS/RI - 91/79.

Bridge, J.S., and Jarvis, J. (1985). "Flow and sediment transport data."

De Vriend, H.J., and Geldof, H.J. (1983). "Main flow velocity in short and sharply curved river bends. Communications on hydraulics Report No. 83-6.

Dietrich, W.E., and Smith, J.D. From "Boundary shear stresses and sediment transport in river meanders of sand and gravel." In *River Mendering*, American geophysical union, Water Resources Monagram. Ikeda, S, and Parker, G. (eds.).

Markham, A.J. (1990). "Flow and sediment processes in gravel bed river bends." PhD. Thesis. Queen Mary and Westfield College, Univ. of London.

Maynord, S. (1990). On behalf of the U.S. Army Engineering Waterways Experiment Station.

Odgaard, A.J., and Mosconi, C.E. (1987). "Streambank protection by submeregbed vanes." *J. Hydr. Engrg.*, ASCE, 113(4), 520-536.

Sources of data for trapezoidal channels.

Odgaard, A.J., and Bergs, M.A. (1988). "Flow processes in curved alluvial channel." *Wat. Res. Res.*, 24(1), 45-56.

Odgaard, A.J. (1984). "Flow and bed topography in alluvial channel bend." *J. Hydr. Engrg.*, ASCE, 110(4), 521-536.

WES. Received from S. Maynard of the Hydraulics Laboratory Structures Division of the US Army Engineering Waterways Experiment Station.

Ippen, A.T., and Drinker, P.A. (1962). "Boundary shear stresses in curved trapezoidal channels." *J. Hydr. Div.*, ASCE, 88(5), 143-179.

Yen, B.C. (1965). "Characteristics of subcritical flow in a meandering channel." Thesis, Inst. for Hydr. Res., Univ. of Iowa, Iowa City, Iowa.

Hicks, F.E., Jin, Y.C., and Steffler, P.M. (1990) "Flow near sloped bank in curved channel." *J. Hydr. Engng.* ASCE 116(1) 55-70.

Sources of data for rectangular canals.

Fox, J.A., and Ball, D.J. (1967). "The analysis of secondary flow in bends in open channels." *Proc. Inst. of Civil Engrs.* 39 Paper No. 7087. 467-475.

Rozovskii, (1961); Kikawa et al, (1976); and Struiksmā et al, (1986) from Johannesson, H. and Parker, G. (1989) "Velocity redistribution in meandering rivers." *J. Hydr. Engrg.*, ASCE, 115(8), 1019-1039.

Onishi, Y., Jain, S.C., and Kennedy, J.F. (1972) "Effects of meandering on sediment discharges and friction factors of alluvial streams." Report No. 141. Inst. for Hydr. Res., Univ. of Iowa, Iowa City, Iowa.

Mc Crea, G.H., and Bray, D.I. (1984). "Effects of cross-sectional shape on open channel bend flow." *Proc. from the 1984 CJCE Annual Conference*, Halifax, N.S. May 23-25 1984 pp.515-529.

de Vriend, H.J., and Koch, F.G. (1977) "Flow of water in a curved open channel with a fixed plane bed." Delft Hydraulics Lab., Delft Univ. of Technol., T.O.W. Report R657-V/M1415-II.

de Vriend, H.J., and Koch, F.G. (1978) "Flow of water in a curved open channel with a fixed uneven bed." Delft Hydraulics Lab., Delft Univ. of Technol., T.O.W. Report R657-VI/M1415-II.

Yen, C.L. (1967) "Bed configuration and characteristics of subcritical flow in a meandering channel.) Thesis presented to the University of Iowa, Iowa City, Iowa, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

APPENDIX B - ADDITIONAL PARAMETERS USED IN MODELING APPROACH

RESEARCHER	RIVER	SITE	BEND NUMBER	SLOPE ($\times 10^{-3}$)	DARCY- WEISBACH f	DISCHARGE (m^3/s)	SINUOSITY
Markham & Thorne	Fall	Reach B	1	1.46	0.27	2.82	1.43
Thorne et al.	Fall	Reach A	1	1.16	0.21	5.4	2.5
Thorne et al.	Fall	Reach A	2	2.14	0.73	4.65	1.64
Thorne et al.	Fall	Reach A	3	1.29	0.19	6.75	1.9
Markham & Thorne	Rodrig	Loughston	1	1.8	0.14	17.63	1.59
Thorne	Fall	Reach 1	1	1.73	2.38	4	2.2
Thorne	Fall	Reach 1	2	1.73	0.25	4	2.2
Anthony	Fall	Reach 4	1	1.32	0.36	4	2.1
Bridge & Jarvis	South Esk	Glen Cova	1	5	2.08	13	1.1
Bhowmik	Kaskaskia	Reach 1	1	0.203	0.085	120.6	1.07
Bhowmik	Kaskaskia	Reach 1	2	0.203	0.077	140.2	1.07
Bhowmik	Kaskaskia	Reach 1	3	0.203	0.0905	122.3	1.22
Bhowmik	Kaskaskia	Reach 1	4	0.203	0.159	86.4	2.59
Bhowmik	Kaskaskia	Reach 1	5	0.203	0.116	95.8	2.67
Bhowmik	Kaskaskia	Reach 2	2	0.117	0.084	101.1	1.08
Bhowmik	Kaskaskia	Reach 2	3	0.117	0.091	105.7	4.36
Bhowmik	Kaskaskia	Reach 2	4	0.117	0.091	102.2	1.23
Bathurst & Thorne	Severn	Maes Mawr	1	1.61	0.093	15.3	1.11
Bathurst & Thorne	Severn	Rickay Bridge	1	1.3	0.049	10.7	1.06
Maynard	Missouri	Browers Bend	1	0.19	0.048	1483.5	1.13
Maynard	Missouri	Snyder Bend	2	0.19	0.044	1492.3	1.07
Maynard	Missouri	Glovers Point Bend	3	0.19	0.052	1435.4	1.21
Maynard	Missouri	Winnabago Bend	4	0.19	0.043	1495.4	1.27
Maynard	Missouri	Upper Omaha Mission	5	0.19	0.037	1602.8	1.06
Maynard	Missouri	Middle Omaha Mission	6	0.19	0.039	1538.2	1.14
Maynard	Missouri	Lower Omaha Mission	7	0.19	0.041	1621.4	1.08
Maynard	Missouri	Upper Monona Bend	8	0.19	0.04	1595.3	1.02
Maynard	Missouri	Lower Monona Bend	9	0.19	0.038	1796.4	1.24
Maynard	Missouri	Blackbird Bend	10	0.19	0.037	1562.6	1.15
Maynard	Missouri	TierVile Bend	11	0.19	0.039	1489.1	1.13
de Vriend & Geldof	Dommel	The Netherlands	1	0.52	0.11	1.3	1.3
de Vriend & Geldof	Dommel	The Netherlands	2	0.52	0.116	1.26	1.05
Dietrich & Smith	Muddy Creek	Wyoming	1	1.4	0.145	0.88	1.07
Odgaard & Mosconi	E. Nishnabotna	Iowa	1	6.8	0.7	123	1.65

RECTANGULAR CHANNELS

RESEARCHER	SITE	BEND NUMBER	RADIUS OF CURVATURE (m)	BEND LENGTH (m)	WIDTH (m)	MEAN DEPTH (m)	DISCHARGE (cumecs)	SINUOSITY	D50 (m)	DARCY WEISBACH f	SLOPE
Fox & Ball	Leeds, UK	1	1.07	3.35	0.31	0.15	0.015	1.565	0.0001		
Rozovskii	USSR	1	0.80	2.51	0.80	0.06	0.012	1.57	0.0001		
Kikawa et al.	Japan	1*	4.50	14.14	1.00	0.05	0.020	1.57	0.0009	0.0491	0.002
Kikawa et al.	Japan	2	4.50	14.14	1.00	0.06	0.025	1.57	0.0009	0.0465	0.002
Kikawa et al.	Japan	3	4.50	14.14	1.00	0.06	0.030	1.57	0.0009	0.0408	0.002
Sruitsma et al.	Delft, Holland	1*	12.00	29.32	1.50	0.08	0.047	1.3	0.0005	0.0974	0.00236
Sruitsma et al.	Delft, Holland	2	12.00	29.32	1.50	0.10	0.062	1.3	0.0005	0.0948	0.00203
Onishi et al.	IIHR	1	8.53	13.41	2.34	0.13	0.164	1.11	0.0003		
Onishi et al.	IIHR	2	9.12	14.32	1.17	0.13	0.082	1.11	0.0003		
McCrea & Bray	Fredrickton, Can	1	3.00	3.14	1.00	0.20	0.060	1.05	0.0001	0.0349	0.0002
McCrea & Bray	Fredrickton, Can	2	3.00	3.14	1.00	0.20	0.060	1.05	0.0001	0.0349	0.0002
de Vriend & Koch	LPM	1	4.25	7.85	1.70	0.17	0.191	1.57	0.0001		
de Vriend & Koch	LPM	2	4.25	7.85	1.70	0.17	0.173	1.57	0.0400		
de Vriend & Koch	Delft Hydraulic Lab.	1	50.00	72.00	6.00	0.25	0.615	1.02	0.0001		
de Vriend & Koch	Delft Hydraulic Lab.	2	50.00	72.00	6.00	0.25	0.600	1.02	0.0001		
C. L. Yen	IIHR	1*	8.53	13.40	2.34	0.12	0.087	1.11	0.0003		
Hicks et al.	Alberta University	A1*	3.66	17.2	1.07	0.08	0.038	3.32	0.0001		
Hicks et al.	Alberta University	B1	3.66	17.2	1.07	0.19	0.039	3.32	0.0001		

* Used to derive upper boundary

NOTE
R = Rough
I = Intermediate
S = Smooth
R = Ripples
D = Dunes
P = Plane
S = Straight
M = Meandering
B = Braided

TRAPEZOIDAL CHANNELS

RESEARCHER	SITE	BEND NUMBER	RADIUS OF CURVATURE (m)	BEND LENGTH (m)	WIDTH (m)	MEAN DEPTH (m)	DISCHARGE (cusecs)	SINUOSITY	D50 (m)	SLOPE	DARCY WEBBACH f	AVERAGE VELOCITY (m/s)	DEPTH-AVE TOE VELOCITY (m/s)
Ogden & Bergs Ogden WES	Int Hyd Res.	1	13.11	41.18	2.44	0.15	0.165	1.57	0.0003	0.00116	0.0674	0.45	0.55
	WES	1	13.11	41.17	2.44	0.10	0.144	1.57	0.0003	0.00104	0.0234	0.59	0.73
	H.L.S.D.	1	15.24	23.93	6.76	0.78	5.484	1.11	0.0381	0.00067	0.0492	1.04	1.36
	H.L.S.D.	2	15.24	35.91	6.70	0.77	5.520	1.28	0.0381	0.00042	0.0222	1.07	1.39
WES	H.L.S.D.	3*	15.24	23.92	6.72	0.77	5.485	1.11	0.0381	0.0025	0.0846	1.06	1.50
	H.L.S.D.	1	8.05	14.06	2.69	0.14	0.215	1.14	0.0127	0.0025	0.0846	0.57	0.73
	H.L.S.D.	2	8.05	14.06	2.69	0.14	0.215	1.14	0.0127	0.0025	0.0846	0.57	0.68
	Univ. Iowa	1	8.53	13.40	2.05	0.10	0.171	1.11	0.0001			0.82	0.89
B. C. Yen	Univ. Iowa	2	8.53	13.40	2.15	0.15	0.215	1.11	0.0001			0.69	0.71
Hyman & Dreibler	MIT	1	1.78	1.86	0.61	0.08	0.017	1.04	0.0001	0.00055	0.0267	0.36	0.42

* Used to derive
upper boundary

NOTE
0.0001 =
R = Rough
I = Intermediate
S = Smooth
R = Ripples
D = Dunes
P = Plains
S = Straight
M = Meandering
B = Braided

Explanation

ANALYTICAL AND EMPIRICAL PREDICTION OF SCOUR POOL
DEPTH AND LOCATION IN MEANDER BENDS

prepared for

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SUMMARY

Bend scour due to bed erosion by curved flow leads to deep pools close to the outer bank in meander bends.

The strength of helical flow which drives bend scour is known to be a function of the geometry of the bend and the hydraulics of flow through the bend. Helical flow, fast near bank velocities and deep scour all tend to de-stabilise the outer bank at a bend. Bendways are often the sites of severe bank attack and retreat as a result. Consequently, a great deal of time and effort is spent on the stabilisation of outer bank lines using dumped stone and riprap.

For a bank stabilisation scheme to be successful it must guard against both the erosive velocity (and shear stress) of the flow and the deep scour of the bed adjacent to the bank. Experience shows that failure is just as likely to occur due to under-estimation and allowance for toe scour than to under-design with respect to velocity.

In an earlier report the authors examined the methods available to predict near bank velocities and bank shear stresses at the outer bank in bendways (Thorne and Abt, 1990). In this report the authors go on to consider the methods available to predict near bank scour depth. Both analytical and semi-empirical approaches are considered. The analytical approaches use existing and available bend flow models developed by John Bridge and by Jacob Odgaard. The empirical approach is based on a statistical analysis of hydrometric data from the Red River in Arkansas and Louisiana undertaken by the first author on behalf of the Vicksburg District, US Army Corps of Engineers.

These predictive techniques are tested using a data set for over 250 bends assembled from a variety of rivers around the world. All flows referred to in the set are high, in-bank flows corresponding to 'formative' discharges in the channel. The data set covers a wide range of sizes of river from laboratory channels to very large alluvial streams. It also encompasses rivers with bed materials ranging from sand to boulders. The rivers display both freely migrating bends with outer banks formed in easily erodible alluvium, constrained bends with resistant outer banks and stable bends with revetments at the outer bank.

The results confirm that there is a close relationship between bend geometry and scour depth that may be characterised by a dimensionless graph with the ratio of bend radius of curvature divided by crossing width (R_c/w) on the x-axis and the ratio of bend maximum scour depth to mean crossing depth (d_{max}/\bar{d}_{bar}) on the y-axis.

Application of the bend flow models produced mixed results. The Bridge model produced wide scatter and generally tended to over-estimate scour depth. The model worked better for long radius bends and errors increased alarmingly as the R_c/w value decreased to about 2. It is recommended that application of this model be restricted to bends with R_c/w greater than 4 and that it be recognised that errors of +50% are common and +100% are possible. Bridge has developed a new model which should do better and will supply this for public use shortly (Bridge, personal communication, 1992).

Odgaard's model consistently under-estimated scour depth. For smaller rivers predicted values were about -50%, but on larger rivers the model crashed. However, unlike Bridge's model, the results were consistent even for the tightest bends and actually did quite well for $R_c/w = 1$. Under-predictions seemed to be due to the difficulty the model had in correctly predicting the entrainment and transport of any sediments coarser than fine sand. If this problem could be overcome, the model has great potential. It should be noted that the version of Odgaard's model used here is being replaced by an up-dated and improved model (Odgaard, personal communication, 1992).

Overall, the empirical method produced the best agreement between observed and predicted scour depths. Practically all predictions fell within +/- 50% of observed values across the whole range of river scales, scour depths, bed materials and bend geometries. The great majority of the predictions fall within a band of +30% to -25%, which is close to being acceptable for engineering purposes.

It is therefore recommended that the empirical equation be further tested and evaluated for use as an aid to scour prediction in bends where more sophisticated methods and models are unavailable or inapplicable. This equation is:

$$(d_{\max}/d_{\text{bar}}) = 2.07 - 0.19 \ln (R_c/w)$$

where,

d_{\max} = maximum scour depth in bend

d_{bar} = mean depth at the upstream crossing

R_c = bend radius of curvature

w = width at upstream crossing

The equation refers to maximum scour during high, steady, in-bank flows. It cannot be used to predict low-flow scour, or scour variation with changing stage during a hydrograph. Also, care must be taken if the equation is used to predict scour associated with flows greater than bankfull stage. If there is significant inter-action between the in-channel and overbank portions of the flow then scour patterns and amounts are not predictable on the basis of in-bank hydraulics alone.

PREFACE

This project was sponsored by the Hydraulics Laboratory at the US Army Engineer Waterways Experiment Station. The project was monitored by Dr Steve Maynard. The work was undertaken partly at Colorado State University and partly at the University of Nottingham, England. Throughout the project the active support of engineers and scientists at WES aided its completion. Essential data for the Red River were supplied by Mr Fred Pinkard, Vicksburg District, US Army Corps of Engineers. Data were acquired, reworked and tabulated by research assistants at Colorado State and Nottingham with great tenacity and skill. The Principal Investigators wish to record their thanks to each of the individuals for their valuable contributions.

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MAIN TEXT

Introduction

Most alluvial rivers have a meandering planform. Such rivers naturally migrate back and forth across their flood plain by a combination of relatively orderly meander loop growth and downstream progression which is interrupted occasionally through abrupt by-passing of acute bends by chute and neck cut-offs. Generally, meander growth and progression occur through retreat of the outer bank and advance of the inner bank of a bend, although in particular cases this pattern may be reversed. The resulting flood plain deposits vary widely in their composition and engineering properties, depending on the depositional environment in which they were produced. Easily eroded meander belt alluvium generated by point bar deposition at the inner bank in bends is interspersed by tougher back swamp deposits from slack water areas and resistant clay plugs in abandoned channels (Fisk, 1943; 1947).

A great deal of engineering work is undertaken each year on rivers of all scales from small creeks up to the Lower Mississippi to curtail and control the lateral activity of meanders. This is an essential component of projects to improve navigation, increase flood capacity, stabilize banklines, decrease flood plain destruction and reduce the downstream dredging requirement of the river. Bank stabilization is achieved using structures made from a wide variety of materials and combinations of materials and treatments. For example, on the Mississippi River upper bank riprap and lower bank articulated concrete mattress (ACM) are used to protect against the erosive attack of the near bank flow, together with bank regrading and sub-surface drainage control to prevent mass failure. On smaller rivers riprap may be used alone, or increasingly, its use in combination with low cost alternatives and vegetation is a common solution.

Bank stabilization assumes particular significance where a levee or an important piece of infra-structure is set back only a short distance behind the bank line. At such locations retreat of the bank cannot be allowed because it would put the levee or structure in jeopardy. Yet despite awareness of the potentially damaging impacts of failure, revetments do still occasionally fail, resulting in retreat of the bank line, destruction of areas of the flood plain and, in some cases, the loss of a section of levee. Riprap failure due to under-design of the size or thickness of the riprap blanket are comparatively rare. More often the cause of failure is scouring of the bend pool adjacent to the outer bank to a greater depth than that allowed for in the design of the revetment. Scour below the toe of the revetment may trigger mass failure of the whole bank (including, or followed by, the levee above) by one a variety of mechanisms. The mechanisms identified as being the most critical include: rotational slip, slab-type collapse and retrogressive flow failure (Turnbull et al., 1966; Torrey, 1988). Consequently, it is important to be able to predict the likely maximum scour depth in a bend accurately when designing revetment.

However, past studies have demonstrated the difficulty of making accurate predictions of the likely magnitude and location of the maximum scour depth in a bend. For example, some studies have indicated that the severity of outer bank pool scour in revetted bends actually increases compared to free, alluvial meanders (Friedkin, 1945; Thorne, 1988). Other studies have concluded that the evidence is equivocal and have gone so far as to suggest that stabilizing the outer bank with riprap may lead to a decrease in scour depth (Harvey and Sing, 1989). It is therefore vital that improved approaches to scour depth prediction in meander bends be developed so that scour potential may be accurately predicted and allowed for in the design of the revetment. Predictions should and perhaps must allow for and include any increase in outer bank pool scour that might follow stabilization of the bank line.

A further problem in scour pool prediction lies in the fact that pool scour is discharge related. As stage rises it is generally found that pools in bends are scoured deeper, with the eroded material being deposited on the shoals at the crossings between bends. On the falling stage at the end of the event, material is re-eroded from the crossings and deposited in the next bend pool downstream. Hence, there is no clear indication of the high stage bed topography once the flood has passed. Observations of scour depth at low flows are relatively straight forward. However, it is much more difficult to make observations of scour depths at high flows and so reliable field data are scarce. Yet, it is usually during high flows that maximum scour occurs and it is during, or immediately after, high flows that most failures occur. This explains why the design of stabilization works is based on a flow of relatively long return period such as the 10 or 20 year flood. In this regard it is desirable that methods be established to extrapolate observations or calculations of scour depth for low or intermediate flows up to higher flow levels.

Objectives

In view of the numerous problems of bank instability encountered and the great expenditure of funds on bank stabilization it is desirable that improved methods be developed to predict the ultimate, maximum scour depth in bends of the meandering rivers. Ideally these aspects of channel geometry should be predicted analytically, from the distribution of flow velocity and boundary shear stress in the bend in general, and close to the outer bank in particular. But at present our understanding of flow processes in bends is incomplete and our ability to simulate the flow and sedimentary processes numerically is limited. Also, existing approaches to flow and sediment modeling are research based and often have data requirements that cannot be met in routine, day-to-day applications. This has led to the development and use of empirical equations and curves based on field and laboratory observation, but lacking a basis in the theory of fluid flow and sedimentation in curved channels.

The technical objective of this project is to examine empirical and analytical approaches to scour depth prediction in the light of increased data availability through river and flume studies and in view of recent advances in bend flow modelling. The specific objectives are:

1. To compile an extensive and reliable data base on bend scour depths in relation to bend geometry, boundary materials and flow hydraulics;
2. To examine the applicability and accuracy of present empirically derived predictors of scour pool depth and location;
3. To test the applicability and accuracy of present analytical models of bend flow as predictors of scour pool depth and location;
4. To identify if the erosion resistance and mass stability of the eroding outer bank plays any identifiable role in affecting scour pool depth;
5. To establish the degree to which scour pool depth in reverted bends increases relative to similar, unreverted bends;
6. To examine the difficulties of extrapolating predictions based on flows of short return period to long period or 'design flow' conditions; and
7. To recommend a best approach to scour pool depth and location prediction for alluvial and reverted bends of natural channels for future research and development.

Review of Possible Approaches to be Adopted

Bend Migration and Bank Failure

Bend migration occurs through bank retreat and usually takes place as a result of erosion of the bank and bed adjacent to the bank by the flow, coupled with periodic mass failures of the bank under gravity. Processes and mechanisms of bank erosion and retreat have been reviewed in detail elsewhere and are not exhaustively reported here (Thorne, 1978; US Army, 1981; Thorne, 1982; Thorne and Abt, 1989; Thorne and Abt 1990).

During failure the slump debris falls, slides, or flows to the toe of the bank under gravity. It is removed from there by the river over a period of days, weeks or months. This is the basal clean-out phase of bank erosion. While in place at the toe of the bank, slump debris tends to stabilize the bank by protecting the intact bank from further flow erosion and by buttressing it against mass failure. Hence, it is the rapidity with which it is removed and the bank re-eroded which primarily determines the long-term rate of retreat of the bank (Thorne, 1978; Lapointe and Carson, 1987). This is the case even in situations where the most obvious mechanism of bank retreat is mass failure rather than direct entrainment by the flow. Removal of slump debris and continued bank erosion depend on the near bank sediment balance between in-coming sediment from upstream and

out-going sediment downstream, which in turn depends on the distributions of near bank velocity and boundary shear stress around the bend. If supply and removal rates are balanced then the elevation of the toe is constant and the bank retreats by parallel retreat. However, if the scour potential of the flow near the bank exceeds the supply of material from bank erosion and failures, the bed is scoured to make up the deficit and elevation of the bed is lowered. Bed degradation has the effect of further destabilizing natural banks and thereby increasing the volume and frequency of mass failures. In this way the sediment flux at the toe is balanced and further scouring is suppressed. But, if the outer bank is stabilized by a structure, mass failures do not occur and the bank is non-erodible. In this case the ultimate scour depth is limited either by the resistance to erosion of the substrate or by the decrease in bed shear that occurs when the pool approaches the hydraulically determined maximum for the given bend geometry. If no resistant materials are encountered and the bed material is not capable of armoring by selective entrainment, then very deep scouring may occur before the bed topography fully adjusts to the imposed shear stresses at the bed.

In this respect, models of flow and sediment processes in bends have highlighted the close links between bed scour and bank erosion (Thorne, 1978; Lapointe and Carson, 1987; Thorne and Osman, 1988; Thorne, 1991). Investigations show the locus of bank erosion and retreat to closely follow that of the scour pool in a bend. Partly this is the case because high flow velocities attack both the bed and bank simultaneously, but also the very existence of a deep scour hole close to the bank is itself a destabilizing factor with regard to mass failure. A deep scour hole increases both the bank height and its steepness, both of which reduce the factor of safety with respect to mass failure. There is evidence that the critical height for mass instability may represent a limiting factor on scour depth in bends with banks formed in weak, alluvial materials (Thorne and Osman, 1988). Conversely, deep scour holes are often associated with strong, cohesive banks. This is particularly the case for resistant outcrops of backswamp or clay-plug materials in otherwise alluvial bends. These outcrops often have significant scour holes associated with them.

These concepts and arguments explain why an increase in scour depth may follow stabilization of a bank through the construction of a revetment. Care must be taken to properly allow for this increased scour in the design of the structure in order to guard against failure by launching. Conversely, scour should not be over-predicted as this leads to over-design of the structure, poor cost effectiveness and wasted money.

The depth of scour is also discharge dependent. At low flows pools at bends tend to fill with material washed off the crossing upstream. At medium flows pools are scoured of this temporarily deposited material, while at the highest in-bank discharges the bed is scoured down further still. This makes it difficult to predict maximum bed scour purely on the basis of bed topography established from low-flow hydrographic surveys. Even if high flow soundings are available, it is

usually still necessary to extrapolate because the data refer to a flow lower than the design flood for the structure.

Studies of bend migration and bank retreat have established that the rate and distribution of bank erosion may be related to two fundamental factors: firstly the flow velocity adjacent to the bank (for example Hasegawa, Parker, Ikeda; see Ikeda and Parker, 1991 for a state of art review) and, secondly, the depth of scour adjacent to the bank (for example, Odgaard; see Odgaard, 1989 for a recent summary). Many researchers see these as alternative approaches and argue that if one of the factors is successful then the other must be wrong. It is now well established that in nature banks may retreat either because of flow erosion by direct entrainment, or because of toe scour and mass collapse followed by basal clean-out. Hence, the two factors of near bank velocity and near bank depth are complimentary to one another and in different circumstances either one may be primarily responsible for controlling the rate and distribution of bank retreat. In this respect near bank velocity and scour depth are recognised not as alternative factors in controlling bank stability and retreat, but are complimentary factors.

Modelling Bend Processes

To translate this qualitative understanding of bend flow and sediment processes into a quantitative and predictive approach depends either on the statistical analysis of empirical data defining the relevant variables, or on the application of analytical models for depth averaged flow in bends. The empirical, or data-based approach must be based on experience from a wide range of types and scales of river. The limits to the data-base set limits to the applicability of the method and to the reliability of the predictions. However, while it is desirable to use as large a data-set as possible, close attention must also be paid to data quality. If spurious or unreliable data are included, these greatly diminish the value of the data-base. Hence, the empirical approach is essentially data-dependent and relies on wide ranging but high quality data. Conversely, to understand the rationale for the selection of the particular models it is necessary to review briefly the basis for two-dimensional bend flow modeling.

The complete equations of motion for flow in bends cannot be solved analytically. Provided that certain simplifying assumptions can be made, then numerical solutions are possible (Smith and McLean, 1982). However, controversy in the academic world centers on the nature of the assumptions which are tenable.

The question the validity of assumptions is best addressed through consideration of the relative importance of each of the terms in the equations for downstream and cross-stream slopes at meander bends. These equations have been derived from the governing equations for the vertically averaged motion through bends by Smith and McLean (1984). The derivation requires that certain assumptions be made, the main one being that shear stresses due to lateral boundary layers can be neglected. Dietrich and Whiting (1989) give the slope equations so derived as:

$$S = \frac{(t_{zs})_b}{rgh} + \frac{\langle u_s \rangle}{(1-N)g} \frac{d\langle u_s \rangle}{ds} + \frac{\langle u_n \rangle}{g} \left(\frac{d\langle u_s \rangle}{dn} - \frac{\langle u_s \rangle}{(1-N)R} \right) \quad ()$$

$$S_n = \frac{-(t_{zs})_b}{rgh} - \frac{\langle u_s^2 \rangle}{(1-N)Rg} - \frac{1}{1-N} \frac{\langle u_s \rangle}{g} \frac{d\langle u_n \rangle}{ds} + \frac{\langle u_n \rangle^2}{2(1-N)Rg} \quad ()$$

where S = downstream water surface slope, S_n = cross-stream water surface slope, $(t_{zs})_b$ = downstream component of total boundary shear stress, r = fluid density, g = acceleration due to gravity, h = flow depth, $\langle u_s \rangle$ = vertically averaged downstream component of fluid velocity, $N = n/R$ (n = cross-stream coordinate, following the channel centerline and positive towards the left bank, R = radius of curvature of the channel centerline), s = downstream coordinate, parallel to the channel centerline and $\langle u_n \rangle$ = cross-stream component of fluid velocity. In a simplified form the equations may be written:

$$S = S_1 + S_2 + S_3 \quad (3)$$

$$S_n = S_{n1} + S_{n2} + S_{n3} + S_{n4} \quad (4)$$

Expressed in words, the first equation states that the downstream slope (S) is the sum of the slope components due to the downstream bed shear stress (S_1), the downstream change in momentum (S_2) and the cross-stream change in momentum (S_3). The second equation states that the cross-stream slope (S_n) is the sum of slope components due to the cross-stream shear stress (S_{n1}), the centrifugal acceleration (S_{n2}), the cross-stream change in momentum (S_{n3}) and a small term produced by substitution of the continuity equation into the equations for fluid motion (S_{n4}). Since S_{n4} is small it can safely be neglected (Dietrich and Whiting, 1989).

These full equations show that the actual downstream slope cannot always be approximated by the Darcy-Weisbach equation which only yields S_1 and which is only applicable to uniform flow (where S_2 and S_3 are zero), and that the actual transverse slope at a bend does not depend only on the centrifugal acceleration, but is also affected by cross-stream shear stress and changes in cross-stream momentum. A complete analysis of these factors is beyond the scope of this proposal, but a few points may be made.

With regard to the downstream slope, S , it is accepted that both S_1 and S_2 are important but two schools of thought exist regarding the significance of S_3 . Members of the first school base their approach on the seminal analysis by Engelund (1974) and hold that the S_3 term is negligible. Notable papers by Bridge (1977; 1984) and by Odgaard (1986; Odgaard and Berghs, 1988) follow this approach and present field and flume data to support their case. The second school base their

approach on a re-examination of the equations of fluid motion undertaken at the University of Washington in the late 1970s by a team of researchers including Dietrich, Dunne, McClean and Smith (see Dietrich et al., 1979; Dietrich, 1982; Dietrich and Smith, 1983; Dietrich et al., 1984; Smith and McClean, 1984). They hold that the S_2 and S_3 terms are of comparable magnitude in most natural rivers which have point bars and in which the local radius of curvature changes continuously through the bend, although the S_3 term may well be small in laboratory flumes which lack point bars and have bends of constant radius. With respect to contrasts between sand-bed and gravel-bed rivers, in a recent comparison reported by Dietrich and Whiting (1989) it was concluded that the ratio of S_3 to S_2 in gravel-bed streams was similar to values in sand-bed streams despite marked differences in curvature and bed grain size. This was taken to demonstrate that bed topography effects, and particularly convective acceleration terms due to topographic steering by the point bar, are the dominant controls on downstream slope at bends.

With regard to the cross-stream slope, S_n , Dietrich and Whiting (1989) used detailed field observations to show that the cross-stream momentum transfer term, S_{n3} , is only negligible if flow at the bend entrance is skewed so that the maximum velocity is already near the outer bank. In most rivers, where consecutive bends are of opposite curvature, this will not be the case and the cross-stream discharge and momentum flux associated with the movement of the core of maximum velocity from near the inner to near the outer bank will be an important factor in determining the cross-stream water surface slope in both sand-bed and gravel-bed rivers.

Approaches Adopted

On this basis it would appear that models based on the Engelund approximations do neglect significant terms and that models based on the full equations of Smith and McLean are preferable. However, derivatives of the Smith and McLean analysis place such heavy requirements on the accuracy and availability of data that they cannot be applied to field situations. For example, to properly account for the effects of convective accelerations in bend flow it is necessary to know the water surface topography in the bend to millimeter accuracy (Anthony, 1987; Dietrich and Whiting, 1989). This would be possible to measure at normal flow, in small rivers, although only at very great expense. However, to use such models to predict the longterm bed configuration in large rivers would not be possible and to deal with high flows of around bankfull discharge in even medium sized watercourses is not feasible with present technology.

Recently, in an evaluation of several bend flow models Markham (1990) concluded that models based on the Engelund approach and neglecting the convective acceleration terms can give reasonable approximations of the main flow and morphological features of meander bends. He found that the main features of bends are predictable, but that in detail every bend is different. This

is the case because each bend is the unique product of the sequence of flow events, approach channel orientation and sedimentary materials encountered in the bed and banks. These factors are stochastic and cannot be predicted deterministically. However, for practical and engineering purposes there is no justification for attempting to predict them, since the fine details of flow and morphology are usually of no consequence to engineering schemes to stabilize the bend. This conclusion has also been reached by even the strongest advocates of the most complex bend flow models, who agree that while "blue skies" and academic studies should strive to be geophysically correct, much simpler models may be safely applied for practical and design work (Dietrich and Whiting, 1989).

The most promising approach is, therefore, to use the latest versions of models developed from the Engelund analysis and having data requirements which can realistically be met in real world situations. In a previous study of outer bank toe velocities in bends Thorne and Abt (1990) applied PC compatible versions of models by Bridge and by Odgaard which fulfil these criteria. They were found to give outer bank velocity predictions which were better than those obtainable from empirical curves provided that close attention was paid to the limits of applicability of the models. The complete findings of that study are not repeated here since the work was undertaken for WES under contract number DACW39-89-K-0015 and is reported in detail in the relevant document (Thorne and Abt, 1990).

The technical approach in this project applies and tests the Bridge and Odgaard models as predictors of scour pool depth. In parallel an empirical analysis of the data base will be undertaken to develop morphometric relationships between independent variables describing the bend geometry and dependent variables defining the location and severity of scouring in the bend pool. A project along these lines was recently successfully completed using data from a 1981 hydrographic survey of the Red River, dealing with the reach between Index, Arkansas and Shreveport, Louisiana (Thorne, 1988). Data from 70 bends was used to produce a regression equation for the prediction of maximum bend scour (d_{max}) on the basis of the mean depth of the approach channel at the crossing upstream of the bend (d_{bar}) and the bend geometry represented by the ratio of bend radius (R_c) divided by width at the upstream crossing (w). The regression equation for the empirical method is:

$$(d_{max}/d_{bar}) = 2.07 - 0.19 \ln (R_c/w) \quad (5)$$

The correlation coefficient was 0.8 and was statistically significant at the 0.01 level of confidence. The coefficient of determination was 0.64, indicating that variation in (R_c/w) was able to account for 64% of the variation in (d_{max}/d_{bar}). The lower limit to the applicability of the equation is an R_c/w value of 2. This is consistent with the observation that the monotonic increase

in scour depth with decreasing R_c/w seems to stop at this value. A complete account of the methodology is available in Thorne (1988).

Data-Base: Sources and Features

Sources of Data

The data used in this study come from a wide variety of types and sizes of rivers, located in different physiographic regions and from different parts of the world. The data were compiled from existing reports and papers supplement by information solicited in letters sent to researchers known to be actively interested in bend flow. Different studies and researchers supplied substantially different types of data. Hence, before embarking on the detailed analysis of the data, it is necessary to review each of the data sources and the nature, scope and features of the material supplied. This may seem unnecessary to readers, but it is in fact crucial. Before using any observed data to derive or test empirical and analytical models it is vital to identify the particular characteristics of the data that may substantially impact the results.

Data-Base Characteristics

Thorne and Abt (1990). The first data used came from Thorne and Abt (1990) and were compiled in a project concerned with the prediction of depth averaged velocity over the toe of the outer bank in meanders. The base data for natural rivers taken from Thorne and Abt (1990) are shown in Table 1. These data were compiled especially to allow application of the Bridge and Odgaard models and so all of the required parameters were already recorded. The data had been screened in the previous study and found to be of high quality and reliability. But the previous application to the prediction of near bank velocity did not involve knowing the maximum scour depth at each bend and so no record of the observed value was made in Thorne and Abt's report. In this study the original reports were re-examined and the maximum scour depths observed at each study bend were extracted and added to the data base. It was noted that as the surveys were based on surveying of a finite number of cross-sections (between three and seven per bend) it could not be guaranteed that the recorded maximum scour depth was actually the maximum observed anywhere in the bend. Also it was noted that measurements corresponded to high, in-bank flows in the rivers rather than to 'design flows' of long return period. The additional data used in this study are also listed in Table 1.

The consistency of the data was checked by computing dimensionless parameters of bend geometry and scour depth. The bend geometry is represented by the ratio of bend radius of curvature (R_c) to width at the crossing upstream of the bend (w). The scour depth is represented by the ratio of bend maximum scour depth (d_{max}) to average depth at the crossing upstream of the bend (\bar{d}). The results are listed together with other calculated parameters in Table 2 and are

plotted in a dimensionless graph in Fig. 1. This graph uses (Rc/w) on the x-axis and (d_{max}/\bar{d}) on the y-axis. Theories of bendflow (Bagnold, 1960), flume studies, engineering experience and observations on natural rivers have shown that bend scour may be up to 3 times the mean depth in the approach channel and that the peak scour usually occurs in bends with Rc/w in the range 2 to 3. Examination of Fig. 1 indicates that these data are consistent with these rules of thumb and on this basis there is no reason to doubt their validity.

Red River Hydrographic Surveys. The second and third sources of data in this study are hydrographic surveys of the Red River between Index, Arkansas and Shreveport, Louisiana. Two surveys were used, those of 1981 and 1969. The Red in this reach is highly mobile, with rapid bank erosion, bend migration and planform evolution. Consequently, the bed topography, bend geometry and planform configuration in 1981 bears very little relation to that in 1969. Due to the changes occurring between these two dates, it may be concluded that the surveys yield essentially independent data sets on the relationship between bend geometry and bed topography. The base data for 1981 are shown in Table 3, which uses data taken from Thorne (1988) together with additional data collected in this study. The depths in this data set are referenced to the water surface profile for the two year flow, which was identified by Biedenham et al. (1987) as a good guide to the channel forming flow for this river. However, no measurements of the discharge corresponding to the two year flow at each bend were recorded by Thorne (1988). Information on the variations in the volumetric discharge for the two year flow along the study reach were supplied by the Vicksburg District, US Army Corps of Engineers. Also, a large amount of additional information on the meander wavelength and channel sinuosity were required. The first PI, working with the research assistants working on this project, used maps and aerial photographs supplied by the Vicksburg District to make measurements of the relevant planform parameters for the study bends. Calculated parameters used in this project are listed in Table 4. The Red River base data from the 1969 hydrographic survey are shown in Table 5. with the derived data listed in Table 6.

The dimensionless scour depth versus the Rc/w plots for the Red River surveys are shown in Figs. 2 and 3. Both graphs show the expected distribution, with relative scour depth increasing as Rc/w decreases, and peaking for Rc/w values of around two. While the maximum relative scour depths for $Rc/w = 2$ to 3 are substantially higher in 1969 than those for 1981, the overall shape of the data cloud is the same. It may, therefore, be concluded that the Red River data are consistent with existing knowledge.

British Gravel-Bed Rivers. The fourth data set comes from a study of stable gravel-bed rivers in the United Kingdom reported by Hey and Thorne (1986). The data extracted from their paper together with information on bend geometry obtained in this study, are shown in Table 7. All of the required data for this study were present except the wavelength and radius of curvature for each individual bend. Wavelengths and bend radii could not be measured in the field, as were

the other parameters in this Hey and Thorne's data set, but were estimated from available 1:25,000 topographic maps. The scale of the maps and size of some of the channels was such that while measuring wavelengths was straightforward, estimating the bend radius was difficult on the smallest rivers. Thus the accuracy of R_c must be reduced in this data set.

It should also be noted that the observed maximum bend scour depths are based on a single cross-section in each bend and may not necessarily represent the actual maximum for the bend. Sections were located in the field specifically with the intention of representing the maximum expression of the pool geometry, however. It would therefore be expected that true maxima be similar to the observed values, but they could be a little larger than those observed in some cases.

The calculated data obtained in this study are listed in Table 8 and the variation of dimensionless scour depth with R_c/w is shown in Fig. 4. This shows the expected distribution, with dimensionless scour increasing as R_c/w decreases and peaking for R_c/w of about two. The highest values of relative depth are rather lower than those in the other data sets, consistent with the fact that single sections do not always intersect the point of deepest scour.

An alternative explanation for the lack of d_{max}/d_{bar} values greater than about 2.5 relates to the geomorphological and sedimentary features of these rivers. The rivers are all gravel-bedded and display a coarse surface layer on the bed. Bed material is known to be transported at high in-bank flows, but only at relatively low rates. The coarse bed surface is called a mobile armor. Many engineers and scientists believe that armoring of this type limits the depth of scour because selective winnowing of the finer material leads to the scour hole becoming covered by a lag layer of coarse sediment. In such cases the maximum scour depth for a bend would be less than that observed in a similar bend with a sand bed, where scour potential was not limited by armoring. Other researchers maintain that in a live-bed with an adequate supply of bed load from upstream, scour depends more on flux imbalance between sediment supply and removal and that particle size does not significantly influence scour depth (Emmett Laursen, personal communication at ASCE conferences).

Geomorphologically, these British rivers are in a post-glacial environment. The bed and flood plain materials through which these rivers flow were laid down after the last ice-age, between about 11,000 and 6,000 years ago. At that time melt water was abundant and the rivers were more powerful than they are today. A case can be made that scour potential in such cases is limited by the fact that the contemporary river has inherited its channel and sediments from a more energetic predecessor. This inference is that present rivers are "underfit" with regard to the geometric and geomorphic features of the channel and that they cannot adjust the channel to suit the current flow regime. However, Hey and Thorne (1986) report regime equations for the British rivers which are very similar to those for other gravel-bed rivers known to be in adjustment (from the USA, Canada, New Zealand) and so this seems unlikely.

It is concluded that the gravel-bed river data are reliable, that the channels are adjusted, stable, alluvial features and that any contrasts to the data from sand-bed rivers may be legitimately attributed to the effects of the coarse bed material in these rivers.

River data from other Researchers. The fifth data set consists of river data supplied by various other researchers in response to letters of enquiry. Letters were sent to 66 individuals who are known to be working on bend flow. Of those who responded positively, only four researchers sent data that were sufficiently complete in a timely fashion. Several data sets lacked information vital to the application of the models. Follow-up letters were sent but replies have so far not been received. Other data sets have arrived later, and too late for inclusion in this report. Many researchers did not respond at all and several more politely declined to share their results until they had exhausted their own analyses. The overall response was disappointing, but not unexpected.

The data, listed in Table 9, cover a wide range of scales of flow. At the largest scale, they include twelve bends from the River Ganges in Bangladesh. Since the data were supplied to us it is not possible to check how it was derived in great detail, hence uncertainties concerning the data are greater than for the other sources. However, all the data are reported to refer to high, in-bank flows and should be comparable to the other four data-sets. Calculated data derived from that supplied are listed in Table 10. Plotting the dimensionless scour depth versus the Rc/w ratio (Fig. 5) shows the expected shape to the data cloud, but with marked differences to the other data-sets. First it is noticeable that all of the bends have rather low Rc/w values. The longest bend has an Rc/w of only 7 and most are below 3. In view of this, deep bend scour is to be expected. Even so, the dimensionless scour depths are still surprisingly high. In one case the maximum bend scour is over eight times the mean crossing depth, In 6 cases it is more than four times the mean depth. These figures are out of line with all the other data collected in this study and this must cast doubt on their validity. Examination reveals that the twelve highest points in Fig. 5 represent the twelve bends on the River Ganges. It seems likely that these data are not directly comparable to the rest. The explanation probably lies in the rather low mean crossing depths of the Ganges for a river of such great discharge ($Q = 29,500$ cumecs). Regime equations for the Indian sub-continent would suggest a crossing width of about 3,000 metres and a mean depth of about 7 metres. The observed mean values for the twelve bends are 7,238 metres and 3.61 metres, respectively. From this it may be concluded that the regime of the lower Ganges is not consistent with that of a single thread meandering channel. The channel is much wider and more shallow and is in fact in the transition zone between meandering and braiding. This is consistent with my field observations that the river displays elements of both meandering and braided planforms. Hence, the data are not a fair test of methods developed for single-thread meandering channels in dynamic regime. The data points corresponding to the lower Ganges have been circled to identify them in Fig. 5 and are not used further in the analysis.

Laboratory Flume Data. The sixth and final data set comprises data from laboratory flume channels with mobile bed materials that have been used to simulate flow and bend scour in river bends. These data are included for interest as it is felt that real river data present a better vehicle than flume channels for tests of practical scour predictors. However, physical modeling using mobile bed sediments demonstrates that flumes can simulate the morphology of real rivers and so the data are included. The data are listed in Tables 11 and 12, and are plotted as dimensionless scour depth versus Rc/w in Fig. 6.

Conclusions

The data set prepared in this study contains data for 257 bends on natural rivers (excluding the 12 on the Lower Ganges). It also contains 8 points for laboratory flumes for comparative purposes. The data have been examined carefully and (with the exception of data for the multi-channel Lower Ganges) are believed to be representative of high in-bank flows in rivers with mobile bed materials and predominantly self-formed channels. It is known that some of the bends have alluvial banks formed in erodible and unstable sediments while others have banks that are formed by less erodible and more stable backswamp, clay plug and lithified materials. Others bends still have banks which have been artificially stabilized and which are therefore non-erodible and fully stable with respect to mass failure. This information will be used later, in the analysis of the role of outer bank materials and stability in affecting scour depth, but the initial analysis is based entirely on flow mechanics.

Analysis of Data

Results of Scour Depth Predictions

The data listed in Tables 1 to 9 were used to apply the analytical models of Bridge and of Odgaard, and the empirical model of bend scour developed by Thorne (1988) for the 1981 hydrographic survey of the Red River. When assessing the results it must be remembered that the empirical model must fit the Red River, 1981 data reasonably well, since it was developed from those data. The other 5 data sets do, however, present a legitimate test of the empirical equation.

Observed versus predicted plots for maximum scour depth for each of the data sets are shown in Figs. 7 to 12. There is considerable scatter evident in all the graphs. Errors in predicted scour depths are plotted against bend geometry in Figures 13 to 18.

Examination of Results

Thorne and Abt (1990). The Bridge, Odgaard and Empirical methods were each applied successfully to the data from a range of rivers contained in the Thorne and Abt data set.

The results, in Figures 7 and 13, show that overall the empirical method did best, with a reasonable agreement between observed and predicted scour depths and errors generally less than $\pm 30\%$ and always less than $\pm 50\%$. Errors tended to be greater for tighter bends, that is at lower R_c/w values. It should be noted that the tightest bends, with R_c/w less than 2 cannot be analysed using the empirical method.

The Bridge model produces predictions which fall around the line of perfect agreement, but which involve considerable scatter. Errors are most pronounced for tighter bends and rise to greater than 200% for the most acute bends. For longer bends the method does much better, with errors generally in the range $\pm 50\%$. Poor performance in short radius bends was a feature of the Bridge model in predicting outer bank toe velocity (Thorne and Abt, 1990) and so this result is not unexpected. However, even for the longer radius bends some overestimates of 80 to 90% occur.

The Odgaard model underestimated bend scour in all but a few cases, with errors as great as 60%. Examination of Figure 7 shows that predictions are quite good for bands with scour depths up to about 7 metres deep, but that they seriously under-estimate the observed depths for deeper rivers. This problem was a direct result of the difficulty of getting the model to run for large rivers. Error messages and warnings associated with the generation of negative depths at the inner bank caused problems in the large rivers. Also, the model did not deal well with coarse sediments. If the bed material was coarser than sand then all scouring was suppressed and the predicted maximum scour depth was close to but a little more than the mean depth in the approach channel. This problem was only overcome by using a sand grain size even when the indicated bed material was coarser. This finding corresponds to earlier experience with this version of the Odgaard model.

Red River Hydrographic Survey 1981. The Red River survey of 1981 provided the data for development of the empirical (regression) method and it is therefore self-evident that the method must work fairly well for those data. Even so, the predictions are quite good, with errors generally less than $\pm 20\%$. Errors are greater for the tighter bends.

The data from the Red River include many tight bends which led to scour over-estimates of up to 100% by the Bridge model. The predictions were better for longer radius bends, with errors being in the range $+ 50\%$ to $- 20\%$ for R_c/w greater than 4. Unfortunately, many bends in practical problems have R_c/w values less than 4.

The Odgaard model does poorly for these data, all which come from bends which are really too long for the model in its present form. The model systematically under-estimates the observed scour depth with errors in the general range of -40 to -60% .

Red River Hydrographic Survey 1969. The second set of data from the Red River produce similar results to the first. The empirical method does best, with predicted values

straddling the line of perfect agreement. Errors are in the range $\pm 30\%$ over quite a wide range of R_c/w values.

The Bridge model is again prone to large over-estimates of scour depth, especially for short radius bends, with errors up to 100%. Performance is markedly improved for less acute bends with R_c/w values greater than 4. For these bends the Bridge model usually under-estimates the observed scour depth by about 20%.

Odgaard's model under-estimates scour by 30 to 80% across the whole range of bend geometries. This is consistent with its performance in the other applications.

British Gravel-Bed Rivers. The fourth data set differs from the others in dealing exclusively with gravel and cobble-bed streams. However, despite this marked contrast in bed materials, the empirical method continues to produce the best agreement between observed and predicted scour depths (Figure 10). Examination of the error distribution (Figure 16) shows nearly all errors to be within the range $+50$ to -25% , over a wide range of bend geometries.

The Bridge model did not work well for the gravel-bed river data. It over-predicted scour depth in most cases (Figure 10), with errors as great as 300% (Figure 16). Similarly to the previous data sets, the most serious errors occurred in the tight, short radius bends with R_c/w less than 4 and some excellent predictions are made for some bends. However, even for longer bends over-estimates of 100% still occur in 7 cases.

The Odgaard model was not applied to the gravel-bed river data. In view of its performance in the previous tests, it was clear that the coarse gravel, cobble and boulder sediments common to all the rivers in this set would confound the model and result in predictions of minimal scouring below the mean depth in the approach channel.

River data from various other Researchers. The fifth data set covered a variety of rivers, but was edited to remove the observations from the Lower Ganges in Bangladesh. The examination of the dimensionless scour depth indicated that the values of 6 to 8 for d_{max}/d_{bar} differentiated these data from the great majority of the other observations of bend scour and channel geometry in single-thread meandering rivers.

The plot of observed versus predicted scour depth (Figure 11) shows that the points for the empirical method scatter around the line of perfect agreement. Predictions are better for the deeper scours, but the method over-estimates the maximum depth in some of the shallower bends, by 100% in one case, but more commonly by about 50%.

The Bridge model over-estimates scour depth in all cases, with errors of at least 50% and greater than 100% in four cases.

The Odgaard model does quite well for bends with scour depths of less than 6 metres, but it then does poorly for the deeper rivers. This performance is in line with experience in the previous applications.

Laboratory Flume Data. Application to the laboratory flume data showed both the empirical and Bridge approaches to be fairly reliable. Points for both methods scatter around the line of perfect agreement in Figure 12, with errors generally in the range of $\pm 20\%$ shown in Figure 18. However, the empirical method did seriously over-predict scour in one case (by 60%) and for the same case the Bridge model also did comparatively poorly, being off by 33%. This is the only case where the Bridge model actually performs slightly better than the empirical method.

Summary data for All Rivers

The separate examination and plotting of the data from the different sources may appear long and time-consuming, but it was essential to establish the consistency of the results across the range of river types, researchers and measurement techniques employed by the different studies. On the basis of the examination of the results, it is clear that the performance of the models is in fact generally consistent across all the data sets and that it is, therefore, safe to compile all the results together in order to produce a comprehensive overview. Summary results are shown for all the data in Figures 19 and 20.

Figure 19 compares observed and predicted scour depths for all 256 bends. The data cover maximum scour depths that range from a few centimetres in flume channels up to about seventeen metres in large rivers. This encompasses the likely scour in all but the world's greatest rivers. With regard to very large rivers, data for the Ganges River had, unfortunately, to be excluded because of their inconsistency with the other observations. Data for selected bends of the Lower Mississippi have already been examined in another study and may be reviewed there (Thorne and Hubbard, 1991).

Generally, the empirical method produces the best overall agreement. The points cluster around the line of perfect agreement and errors appear to be randomly distributed. Figure 20 shows that practically all the predictions fall within $\pm 50\%$ of the observed values across the whole range of scour depths and bend geometries, and the vast majority fall within a band from $+30$ to -25% . This compares very favourably to the analytical predictions.

Bridge's model produces wide scatter, with a general tendency to over-estimate scour depth. Examination of the distribution of errors a function of bend geometry confirms the earlier results of Thorne and Abt (1990) in that errors increase markedly as the R_c/w value decreases towards a value of 2. It has been recognised that in such tight bends the flow may strike the outer bank at a steep angle, driving reversed flow upstream of the apex. In this respect, attack of the outer bank is

associated with impinging flow, and this has found to be an intractable problem in both single thread and braided rivers.

Based on these findings, the applicability of the Bridge model is limited to relatively long radius bends, with R_c/w ratios greater than about 4. Even in these bends over-estimates of 50%, and occasionally as much as 100%, must be expected.

Odgaard's model systematically under-predicts scour depth. For the smaller rivers points trend around a line of -50%, but the model crashed and produced little further increase in predicted scour depth once the observed depth passed about 10 metres. However, Figure 20 shows that Odgaard's model was consistent even for the tightest bends, and actually did well for cases where R_c/w was around unity. If the problems of bed material mobility and application to large rivers can be solved, then Odgaard's model has real potential, especially for tight, short radius bends where the empirical model is inapplicable and the Bridge model is inaccurate.

Conclusions

This study has assembled a fairly large and internally consistent data-set for scour at bends. Application of three different approaches to scour prediction has illustrated the present problems of making accurate predictions in bends of different geometry and size. The empirical method produces the best overall agreement, but for longer radius bends Bridge's model does nearly as well. The empirical method cannot be used for tight bends with $R_c/w = 2$ being a lower boundary. Although it does produce predictions, Bridge's model should not be used for shorter radius bends, where it seriously over-predicts scour depth. Odgaard's model remains consistent in under-estimating scour depth by about 50% over a wide range of bend geometries. Accuracy actually improves for very tight bends. The model, therefore shows promise, but could not be used routinely in its 1988 version.

It should be noted that both Bridge and Odgaard are continuously developing and upgrading their models. I understand from both researchers that the problems and difficulties noted in this study are consistent with their experience and that they have been addressed in subsequent editions of the models. The latest versions will be supplied to me to replace the earlier ones as soon as they are ready, but nothing has appeared as yet (May 1992). Updated predictions will be produced when the new versions arrive.

Scour in Revetted and Free Meanders

It was discussed in the Introduction that there are reasons for thinking that scour depths in revetted bends are deeper than for free meanders of similar geometry. The primary purpose of this study is to assess the capabilities of the models to predict scour in revetted bends, and so this

part of the analysis concentrates on bends with revetted outer banks. However, as the data-set includes both revetted and free meanders, the bends were split along those lines, to allow comparison of revetted and free conditions. Bends which were wholly or partially constrained by resistant outcrops of clay, backswamp deposits or Pleistocene materials were excluded. Such bends would be expected to have scour depths intermediate between revetted and free states, and so a clearer picture should emerge.

The observed and predicted results for revetted bends are shown in Figure 21. They are broadly in line with the overall study. Bridge's model produces the most scatter and generally over-predicts scour, Odgaard's model systematically under-predicts and the empirical model does comparatively well. Examination of the distribution of errors (Figure 22) shows how errors in the Bridge model increase for tight bends. His model looks inapplicable to revetted bends with Rc/w less than 4. But for longer bends the model does quite well, with predictions in the approximate range +15 to - 25%. The empirical model is successful, but as a considerable number of the revetted bends come from the Red River, 1981 hydrographic survey, this is not a stringent test for the method. Errors are evenly distributed from a range of Rc/w values and fall in the range +25 to - 15%. Odgaard's model seriously under-predicts scour in all cases.

In the Red River study of Thorne (1988), a separate empirical analysis was developed for revetted bends. Like the general equation for all bends, this was based on the logarithm of $((Rc/w) - 2)$ as the x variable. The least squares regression equation was:

$$(d_{max}/\bar{d}) = 2.15 - 0.27 \ln ((Rc/w) - 2) \quad (6)$$

To examine if this equation had potential for the expanded data-set a semi-log plot of $((Rc/w) - 2)$ versus d_{max}/\bar{d} was plotted (Figure 23). The scatter shows a linear trend to the data with a negative slope. Consequently, the data are consistent with the form of the empirical revetment scour equation. Figure 24 shows the observed and predicted scour depths for the empirical and revetment equations. Figure 25 shows the distribution of errors as a function of bend geometry. There is no obvious improvement in prediction for the revetment equation, and it actually does less well in the revetted long radius bends of the Missouri River. On the basis of this test it does not appear that the equation for revetted bends is any better than that for all bends.

Figure 26 shows a semi-log plot for the free meanders. Comparison with the revetted bends in Figure 23 shows that the two data clouds overlap to such a degree that there is no easy way to discriminate between them. This suggests that the empirical analysis based on a semi-log equation is equally applicable to bends of all outer bank types. This finding is in contrast to the findings of the earlier study on the Red River. There are at least two explanations for this. Firstly, it comes about because the wider range of conditions in the rivers of the large data set have swamped differences due to bank condition. Thus, while bank effects may be identified on a particular river,

they cannot be discerned in a data-set for many different rivers. Secondly, the parameters used in this analysis may tend to collapse data for different bank conditions together. This is the case because the width and mean depth at the crossing are not themselves independent of the bank condition. Rivers with stiff banks tend to be narrower and deeper than those with erodible banks. Hence, in using crossing width and mean depth to scale bend geometry and scour depth, the bank type effects are being implicitly taken into account. Consideration of the impacts of revetments on the regime geometry would help to resolve this issue, but is getting beyond the scope of this study.

It can be concluded though that the empirical method constitutes a robust predictor of scour depth for bends of a variety of bank types and conditions, with errors usually within the range $\pm 30\%$.

Extrapolation to High Flows

The flows referenced in this report are all high, in-bank flows that are close to 'bankfull' discharge. In geomorphological terms, they are believed to be the channel forming flow for alluvial rivers: that is the flow to which the major morphological and geometrical features are adjusted. That is not to say that higher flows do not have significant impacts on the channel and the riparian zone. It is well known that scour of both in-channel pools and overbank areas can be considerable during flood flows. However, over longer periods of say 20 to 50 years, the impacts of high magnitude, low frequency events are not as important in forming the landscape as the impacts of flows at and around bankfull flow.

In the case of revetment design, it is essential that the structure be able to withstand the short term impact of high magnitude events, as well as being in sympathy with the hydraulic geometry and fluvial morphology of the river, which are products of lesser events. Thus, although the use of flows around bankfull have a role in determining the alignment and orientation of the bank line, the strength and scour protection must guard against short duration but higher flows. The highest flow to be guarded against is termed the 'design discharge' and will usually be a flow of quite long return period. Depending on the level of acceptable risk and the value of the land being protected, return periods of 20 or even 50 years may be used.

In terms of application of the scour predictors tested here this poses a problem because the flows referenced have return periods of the order of 1 to 5 years.

In order to solve this problem it is necessary to know something about the flow stage associated with the design flood in relation to the elevation of flood plain and, more particularly, the top of the revetment. In general terms, the stage may be classed as either: at or below top bank; slightly overbank; or significantly overbank.

For in-bank or bank top flows little difficulty arises. The analytical and empirical methods described here are specifically designed to deal with high in-bank flows under which the bed is highly mobile and to which the geometry of the channel is adjusted. In this respect the design flow should be ideal for application of the methods. Where the design flood does not overtop the revetment and is contained within the channel, no extrapolation should be necessary.

Design flows which are slightly out of bank pose more of a problem, since by strict definition they are beyond the scope of the models and methods presented here. This is the case because once the flow goes out of bank strong eddies develop at the inter-face between the channel and flood plain elements. These eddies induce water surface topography, momentum exchanges and shear stress distributions which cannot be predicted on the basis of in-bank flow models alone. However, provided that the depth of flow on the flood plain is small, it may be possible to extrapolate the approach for in-bank flows up to out of bank conditions. This is the case because relatively high flood plain roughness and low flood plain flow depth combine to produce low

velocities of flow in comparison to those in the channel. Hence, the overbank area may be considered more as a reservoir than an extended channel. Under these circumstances, water flow tends to follow the line of the channel, with stable boundaries between the flowing in-bank and static, overbank water. A notional line extending up from the bank top to the water surface may be used to delineate the channel for modelling purposes and the methods described here may be applied using the notional geometry of the cross-section and planform of the flowing channel. Experience suggests that this type of extrapolation may be acceptable where the depth of flow over the flood plain is less than 20% of the channel depth.

When the depth of flow over the flood plain is a significant proportion of that in the channel there is strong inter-action between the in-channel and overbank elements. The flood plain has ceased to be a reservoir and become instead the flood way of a two-stage channel, quite possibly carrying the bulk of the discharge. There is active exchange of water (and sediment) between the channel and the flood plain flows, and the in-bank element cannot be considered in isolation. For example, recent research in the Science and Engineering Research Council's flood flume facility at Wallingford has shown that when the sense of rotation of the main, helical flow in a bend may be reversed when the overbank flow depth attains 50% of the channel depth (Donald Knight, personal communication, 1991). This is associated with the meandering channel acting as a slot in the bed of the floodway rather than as a sinuous channel in its own right.

Under such circumstances it is unrealistic to expect any channel flow model to accurately predict scour pattern or depth based only on the in-bank flow parameters. Major scour will probably be associated with points where overbank flow plunges back into the channel, and this may well be at the inner bank, or at some indeterminate point controlled by the pattern of flood plain topography, vegetation or development. With the present state of art either physical or numerical modeling of the flow in the entire flood plain/channel system would be required to make reliable predictions of velocity and scour patterns.

CONCLUSIONS AND RECOMMENDATIONS

This study has assembled a data-set for 256 bends in single-thread rivers, stream and flume channels. The data have been examined and screened and are believed to be of a high quality. All the observations correspond to high in-bank flows and should represent the channel forming flow. All the rivers are believed to be flowing through alluvial channels in as much as the bed materials are fully mobile for the reference conditions. On this basis the data should allow application of the models for bend flow and scour prediction.

The results indicate that considerable errors can occur in the predicted scour depths. The empirical method produces the most reliable scour estimates, with the great majority of predictions

being within +/- 25% of the observed maximum bend scour depth. Bridge's bend flow model does nearly as well for long radius bends, but massively over-predicts scour depth for tight bends. As a general rule, the Bridge method should only be used for bends with Rc/w ratios greater than 4. The Odgaard model was the least satisfactory of the three. It systematically under-predicted scour depth by about 50% and failed to account for the deep scour holes found in large rivers. Problems with the model center on the entrainment function which does not predict any scour in bed materials coarser than sand, and in large rivers where the program crashed due to the prediction of negative depths at the inner bank.

There was no evidence that revetted bends suffer deeper scour than free meanders, at least in terms of the dimensionless scour depth (d_{max}/d_{bar}) scaled on the mean depth at the upstream crossing. This may be because the mean crossing depth is itself a function of bank stiffness, and so bank effects are accounted for in its use as a scour scaling parameter. If true, this further strengthens the range of applicability of the empirical approach adopted by Thorne (1988).

Extrapolation of the approaches tested here to higher flows is limited by the inter-action between in-bank and out of bank components of the flow for overbank events. Where this is negligible, notional walls may be envisaged, separating the channel flow from the flood plain storage and allowing the delineation of geometrical and hydraulic parameters for the in-channel portion of the flow. Hence, provided that the water level associated with the design flood is only slightly above the top of the revetment and remains mostly in-bank, then the methods described here can be used with caution. However, when the overbank flow depth is a significant proportion of the channel depth, the import and export of water from the channel to the flood plain is not negligible. Flow patterns and bed scour will be dominated by water and momentum fluxes which cannot be predicted by these (or any other) in-bank flow models and it would not be wise to attempt to extrapolate the methodology to such flows. If significant out-of bank flows over and behind the revetment are to be allowed for in the design then some form of flood plain water and sediment model is required.

REFERENCES

- Anthony, D.J. (1987) "Stage dependent channel adjustments in a meandering river: Fall river, Colorado" Unpublished Masters Thesis, Colorado State University, Fort Collins, Co, 180p.
- Bridge, J.S. 1977. 'Flow, bed topography, grain size and sedimentary structure in open-channel bends: a three-dimensional model', *Earth Surface Processes*, 2, 401-416.
- Bridge, J.S. 1984 "Flow and sedimentary process in river bends; comparison of field observations and theory" In, *River Meandering*, C.M. Elliott (ed.), ASCE, New York, USA, 857-872.
- Dietrich, W.E., Smith, J.D. and Dunne, T. 1979 "Flow and sediment transport in a sand-bedded meander" *Journal of Geology*, 87, 305-315.
- Dietrich, W.E. 1982 "Flow, boundary shear stress and sediment transport in a river meander" Ph.D dissertation, Univ. Washington at Seattle, 261p.

Dietrich, W.E., Smith, J.D. and Dunne, T. 1984 "Boundary shear stress, sediment transport and bed morphology in a sand-bedded river during high and low flow" In, *River Meandering*, C.M. Elliott (ed.), ASCE, New York, USA, 632-639.

Dietrich, W.E. and Whiting, P. 1989 "Boundary shear stress and sediment transport in river meanders of sand and gravel" In, *River Meandering*, S. Ikeda and G. Parker (eds.), American Geophysical Union, Water Resources Monograph 12, 2000 Florida Avenue, Washington D.C., USA, 1-50.

Engelund, F. 1974 "Flow and bed topography in channel bends" *Journal of the Hydraulics Division*, ASCE, 100(11), 1631-1648.

Fisk, H.N. (1943) "Geological Report on Clay Plugs in the Vicksburg Engineer District: Unpublished report, Mississippi River Commission, 19p.

Fisk, H.N. (1947) "Fine-grained alluvial deposits and their effects on Mississippi River activity" Waterways Experiment Station Report, Vicksburg, 82p.

Friedkin, J.F. (1945) "A laboratory study of the meandering of alluvial rivers" Waterways Experiment Station, 40p.

Harvey, M. D. and Sing, E. F. (1989). "The effects of bank protection on river morphology." In *Hydraulic Engineering*, M. A. Ports (Ed.), Proc. 1989 National Conference on Hydraulic Engineering, ASCE, 212-217.

Lapointe, M.F. and Carson, M.A. (1986) "Migration patterns of an asymmetrical meandering river: The Rouge River, Quebec." *Water Resources Research*, 22, 731-743.

Markham, A.J. 1990. '*Flow and Sediment Processes in Gravel-Bed River Bends*', thesis submitted to the Department of Geography, Queen Mary and Westfield College, London, in complete fulfilment of the requirements of the degree of Doctor of Philosophy, 419p.

Odgaard, A.J. 1986. 'Meander flow model. II: Applications', *Journal of Hydraulic Engineering*, 112 (12), 1137-1150.

Odgaard, A.J. and Bergs, M.A. 1988 "Flow processes in a curved alluvial channel" *Water Resources Research*, 24(1), 45-56.

Smith, J.D. and McLean, S. R. 1984 "A model for meandering streams" *Water Resources Research*, 20(4), 1301-1315.

Thorne, C. R. (1978). "Processes of bank erosion in river channels." *unpublished Ph.D. thesis*, Univ. of East Anglia, Norwich, NR4 7TJ, England, 447p.

Thorne, C R (1982) "Processes and mechanisms of River Bank Erosion", in *Gravel-Bed Rivers: Fluvial Processes, Engineering and Management*", R D Hey, J C Bathurst and C R Thorne (eds), J Wiley and Sons, Chichester, England, pp 227-271

Thorne, C R (1988) "Bank processes on the Red River between Index, Arkansas and Shreveport, Louisiana" Final Report to the US Army European Research Office under contract number DAJA45-88-C-0018, Dept. Geography, Queen Mary College, December 1988, 50p

Thorne, C.R. and Abt, S.R. (1989) "Bank Erosion Modeling and Assessment Techniques" Final Report to the US Army Engineers Waterways Experiment Station, under contract number DACW39-87-D-0031, Colorado State University, Ft Collins, Co., November 1989, 4 Parts.

Thorne, C R & Osman, A M (1988) "Riverbank Stability Analysis: II Applications" Journal of Hydraulic Engineering, ASCE, Vol 114, No 2, pp 151-172

Thorne, C.R. and Abt, S.R. (1990) "Estimation of Velocity and shear stress at the outer bank in river bends" Final Report to the US Army Engineers Waterways Experiment Station, under contract number DACW39-89-K-0015, Colorado State University, Ft Collins, Co., May 1990.

Thorne, C.R. (1991) "Bank erosion and meander migration of the Red and Mississippi Rivers, USA" in Proceedings of the XX General Assembly of the IAHS, Vienna, Austria, 11-24 August, 1991.

Thorne, C.R. and Hubbard, L.J. (1991) "Flow and bed topography in the Mississippi River" Report to the Mississippi River Commission (in preparation).

Torrey, V.H. (1988) "Retrogressive failures in sand deposits of the Mississippi River" Waterways Experiment Station, Technical Report GL-88-9.

Turnbull, W. J., Krinitzsky, M. and Weaver, F. J. (1966). "Bank erosion in soils of the lower Mississippi Valley." *J. Soil Mech. Div.*, Proc. ASCE, 92(SM1), 121-136.

U.S. Army Corps of Engineers. (1981). "The stream bank erosion control evaluation and demonstration act of 1974, Section 32, Public Law 93-251. Appendix F - Yazoo River Basin demonstration projects." Final report to congress, Office of the Chief of Engineers, Washington DC., USA.

DIAGRAMS AND TABLES

Fig. 1 Dimensionless Maximum Bend Scour Depth as a function of Bend Geometry for Data from Thorne and Abt (1990)

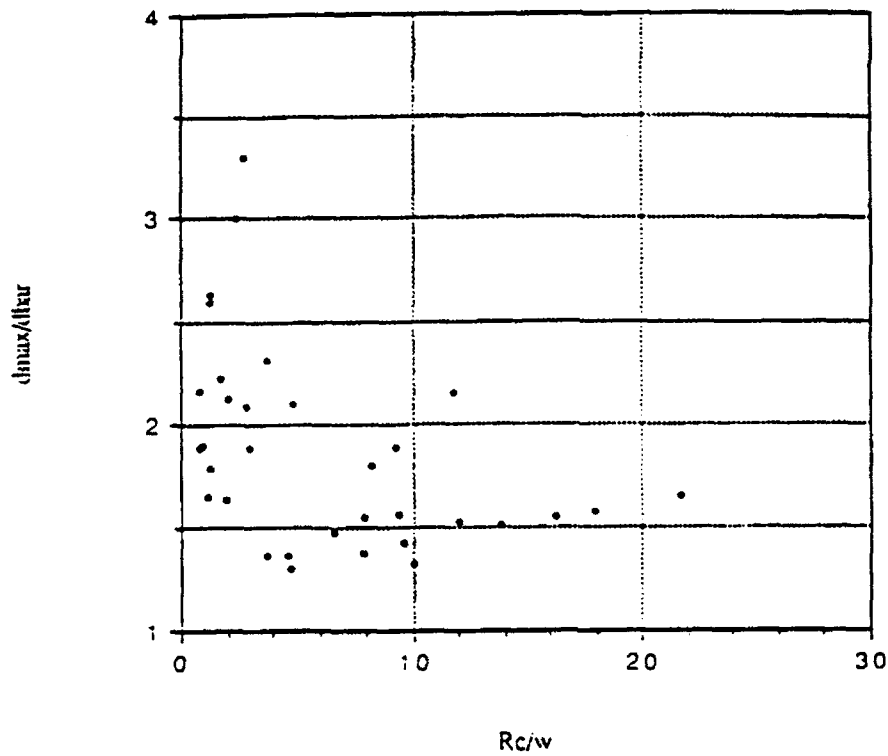


Fig. 2 Dimensionless Maximum Bend Scour Depth as a function of Bend Geometry for Red River, 1981

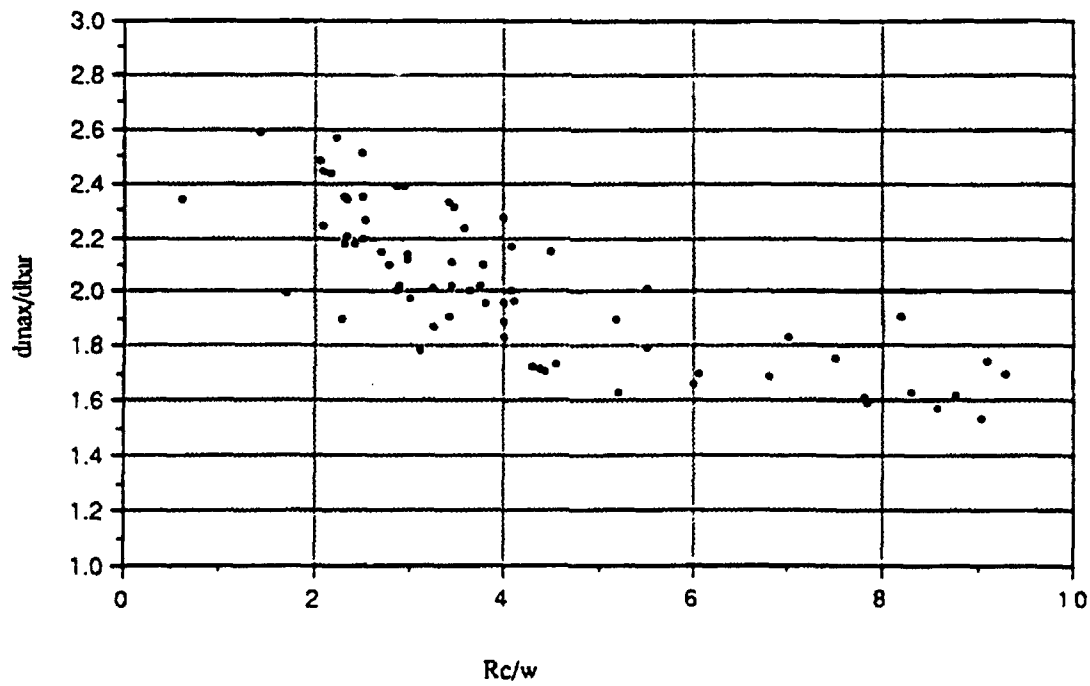


Fig. 3 Dimensionless Maximum Bend Scour Depth as a function of Bend Geometry for Red River 1969

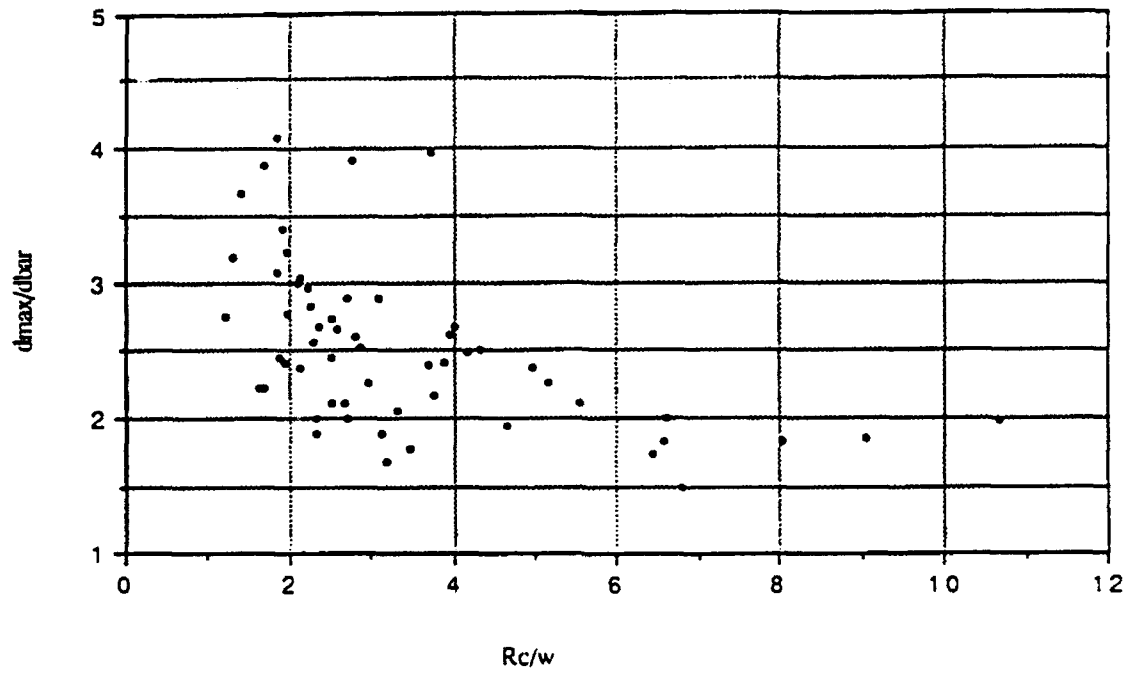


Fig. 4 Dimensionless Maximum Bend Scour Depth as a function of Bend Geometry for British Gravel-Bed Rivers

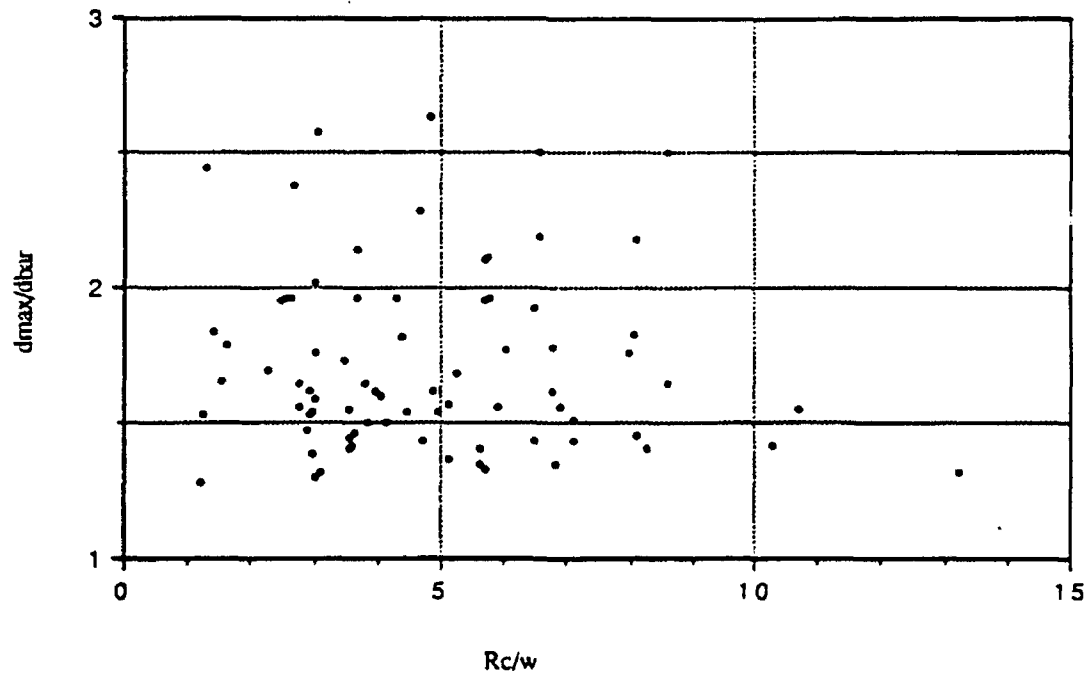


Fig. 5 Dimensionless Maximum Bend Scour as a function of Bend Geometry for data from various researchers Note: open symbols from River Ganges

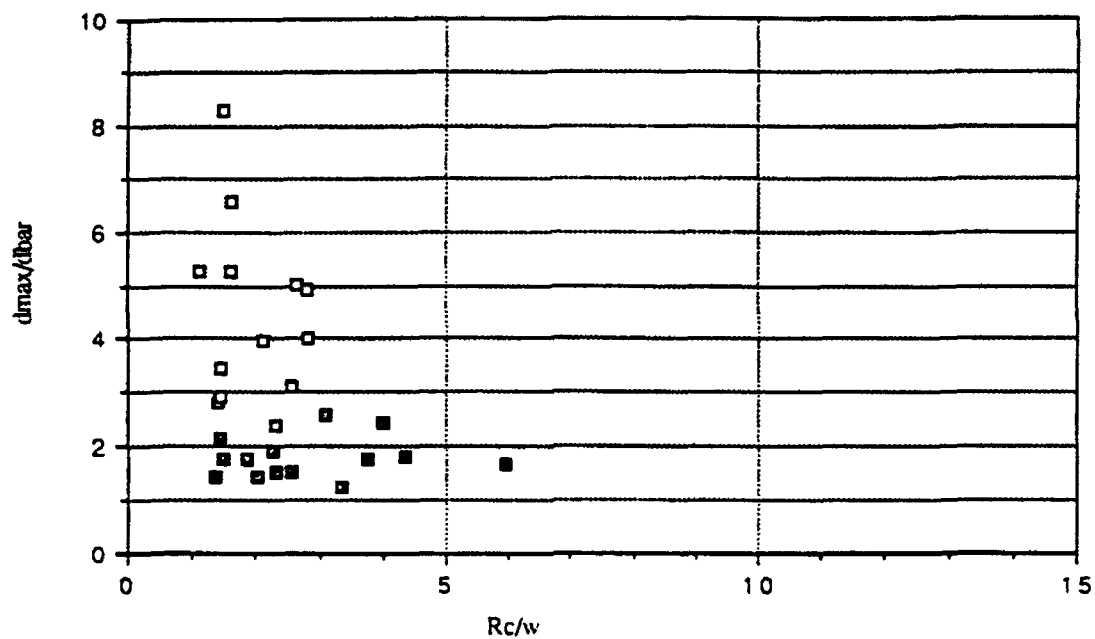


Fig. 6 Dimensionless Maximum Bend Scour Depth for Flume data

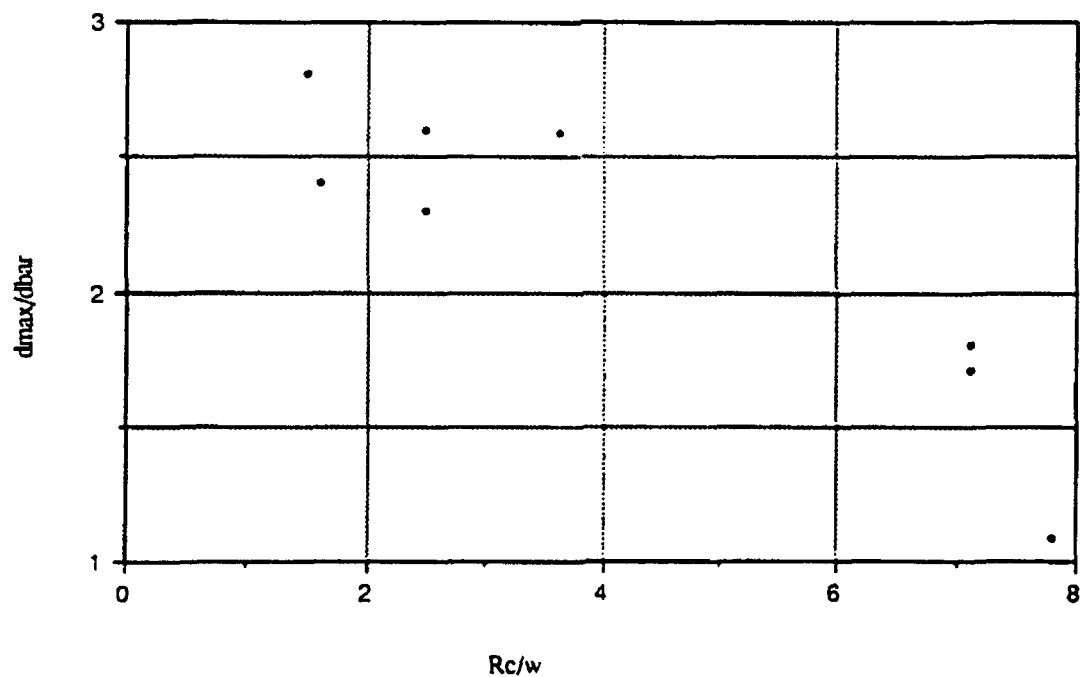


Fig. 7 Observed vs Predicted Maximum Bend Scour Depths
for Data from Thorne and Abr 1990

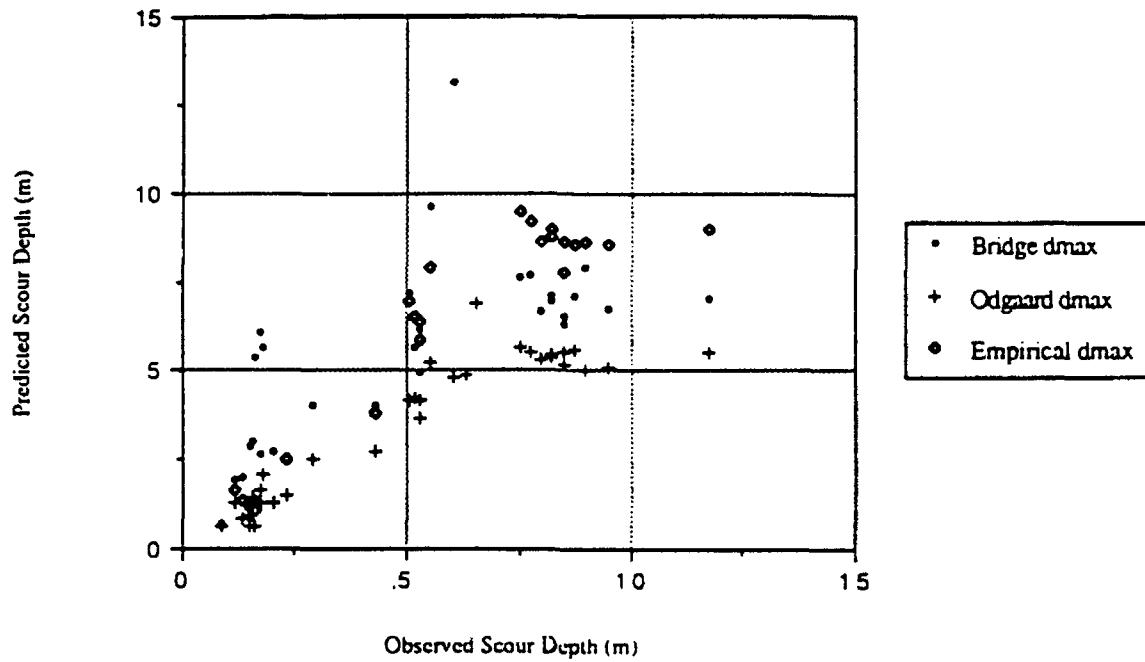


Fig. 8 Observed vs Predicted Maximum Bend Scour Depths for
Data from Red River 1981(Thorne, 1988)

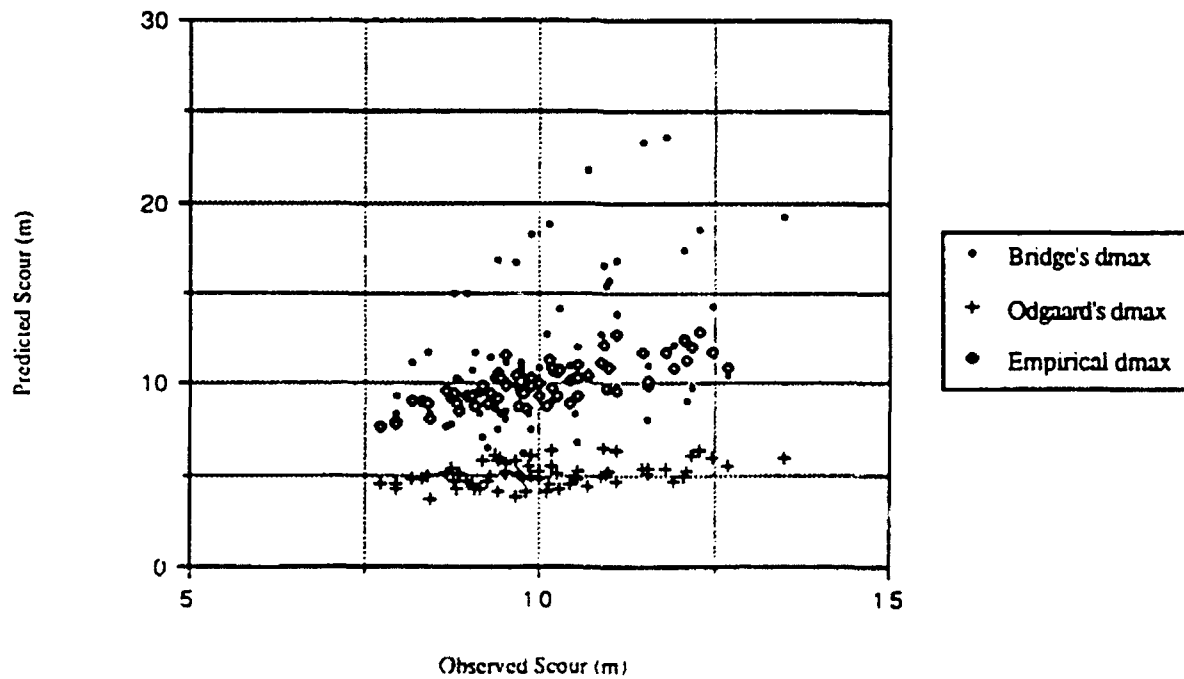


Fig. 9 Observed vs Predicted Maximum Bend Scour Depths for Data from Red River 1969

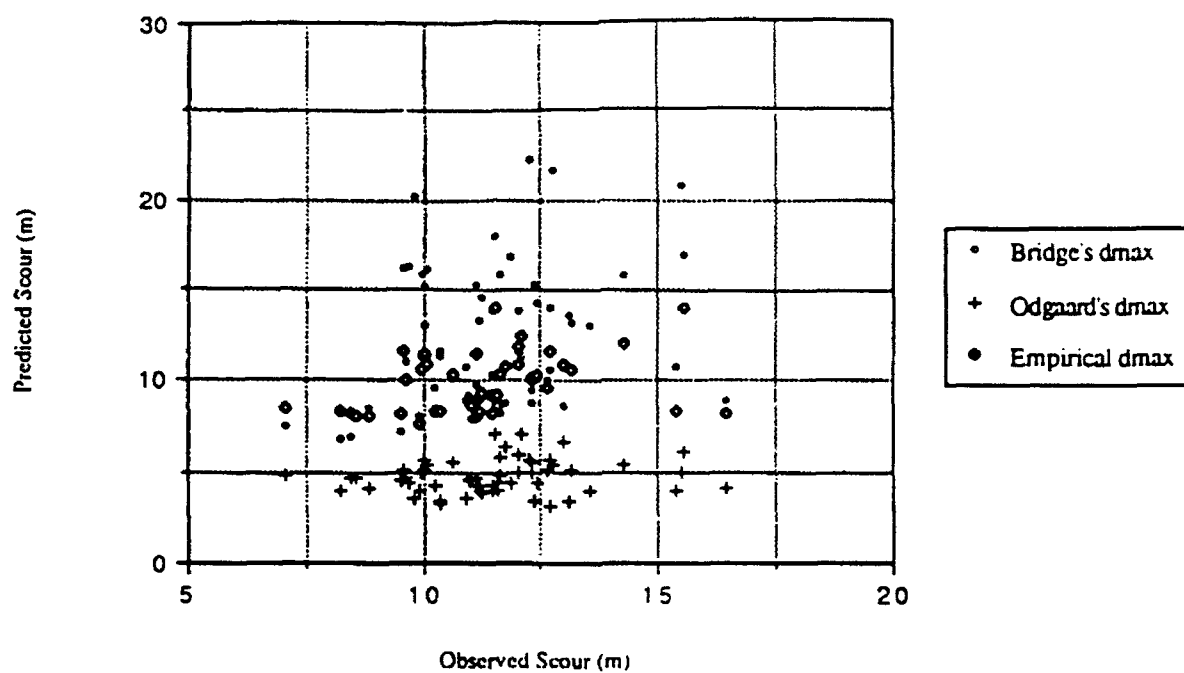


Fig. 10 Observed vs Predicted Maximum Bend Scour Depth for British Gravel-Bed Rivers

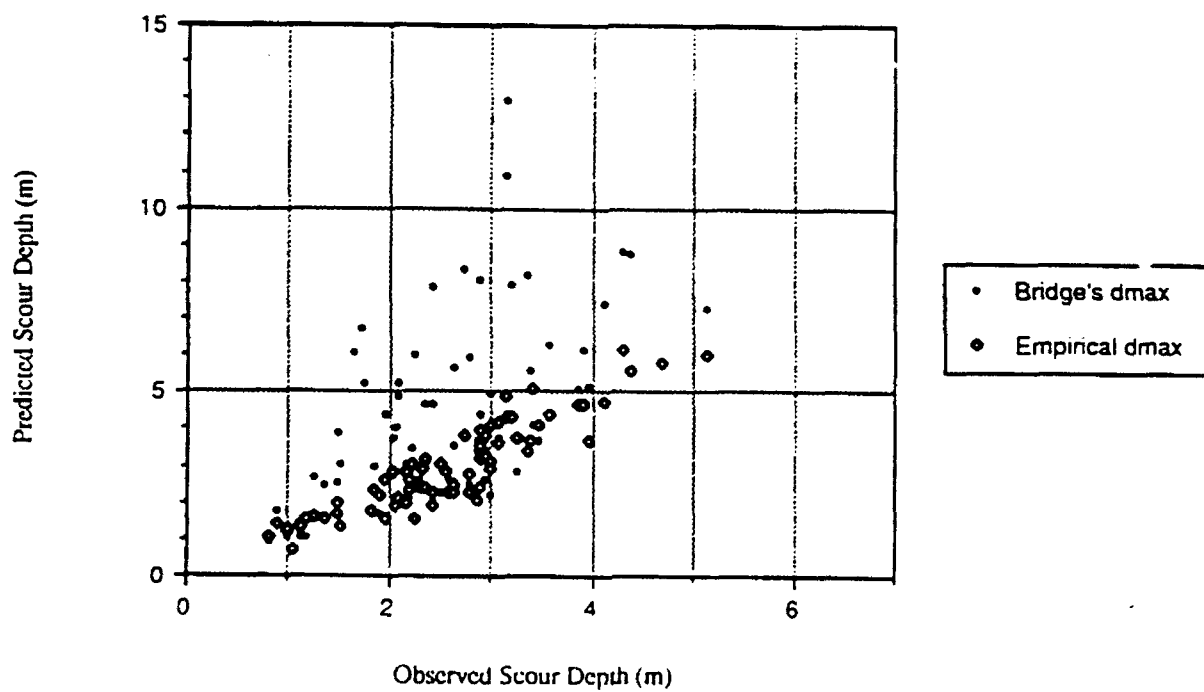


Fig. 11 Observed vs Predicted Maximum Bend Scour Depths for data from various researchers

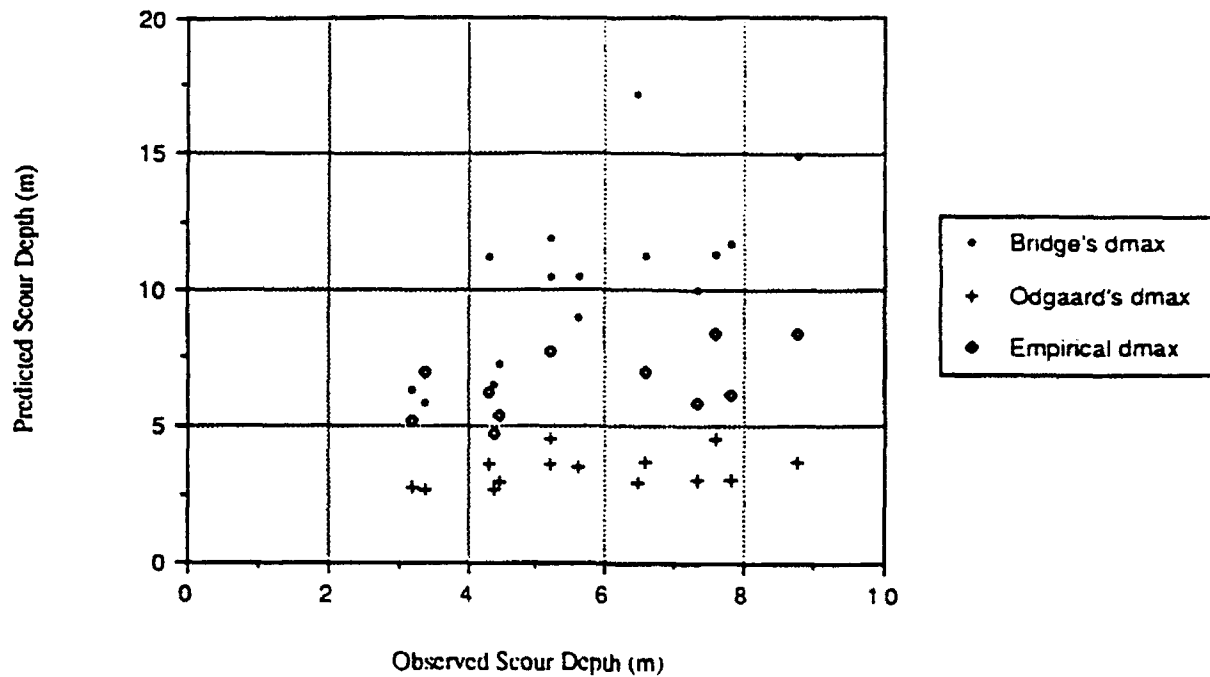


Fig. 12 Observed vs Predicted Maximum Bend Scour Depths for Flume Data

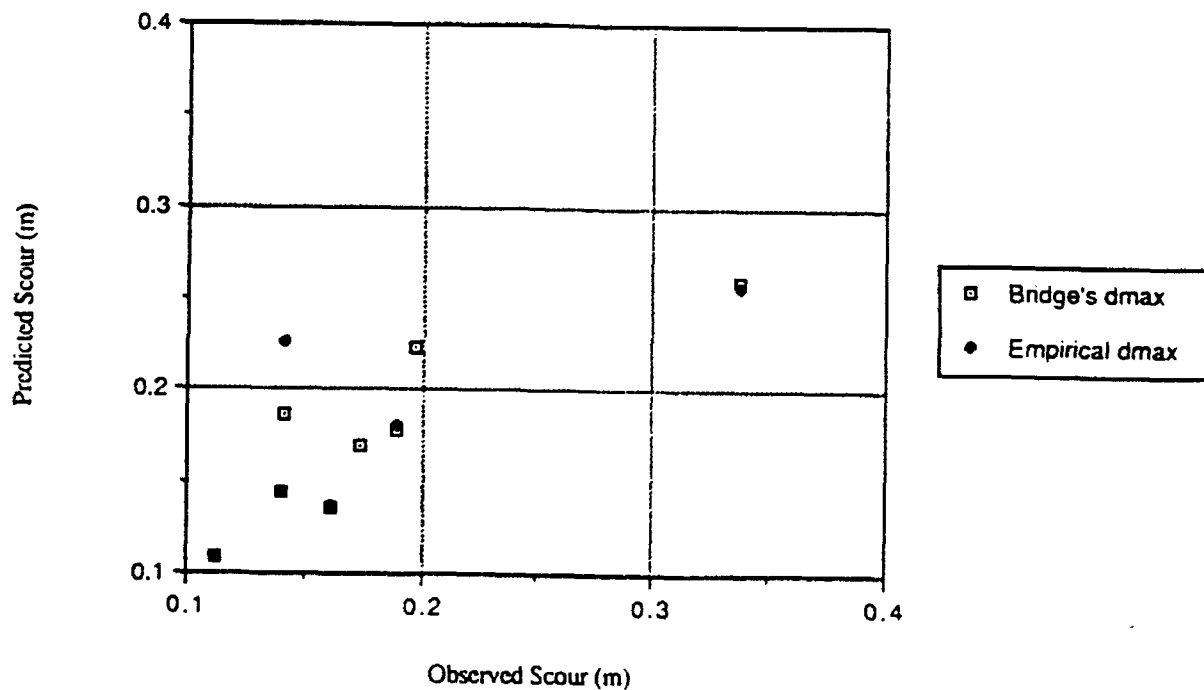


Fig. 13 Errors in Predicted Maximum Bend Scour Depth for Natural River data from Thorne and Abt (1990)

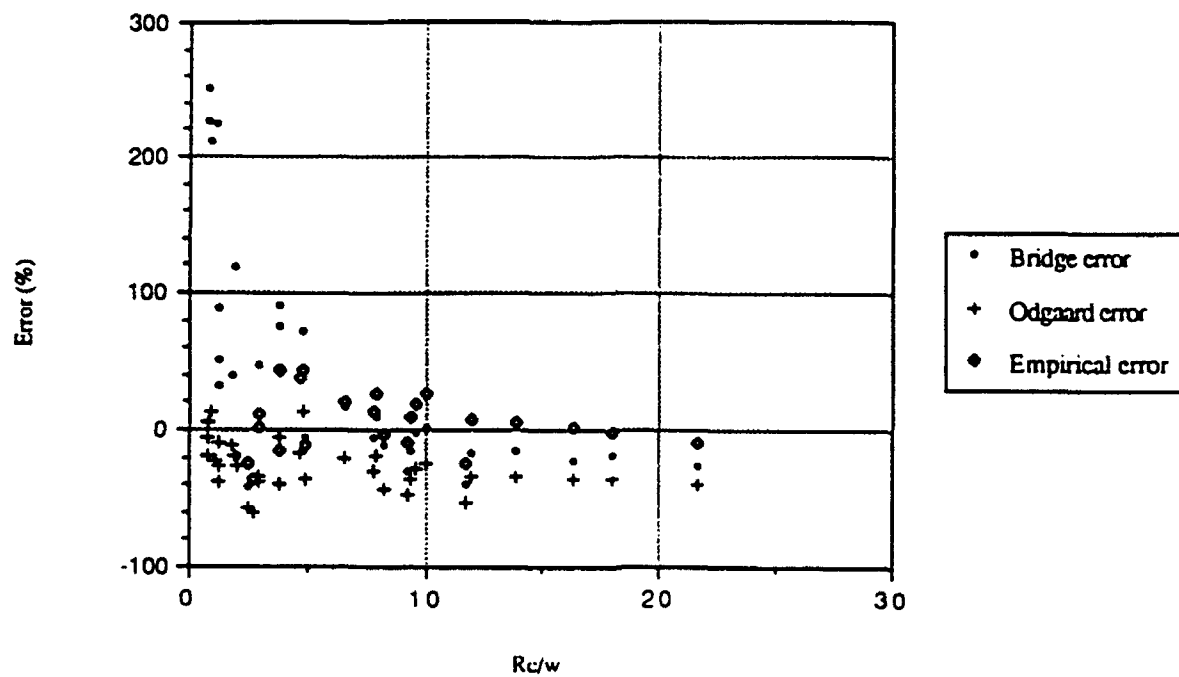


Fig. 14 Errors in Predicted Maximum Bend Scour Depths for Red River 1981 data
(Note: empirical method was developed from these data)

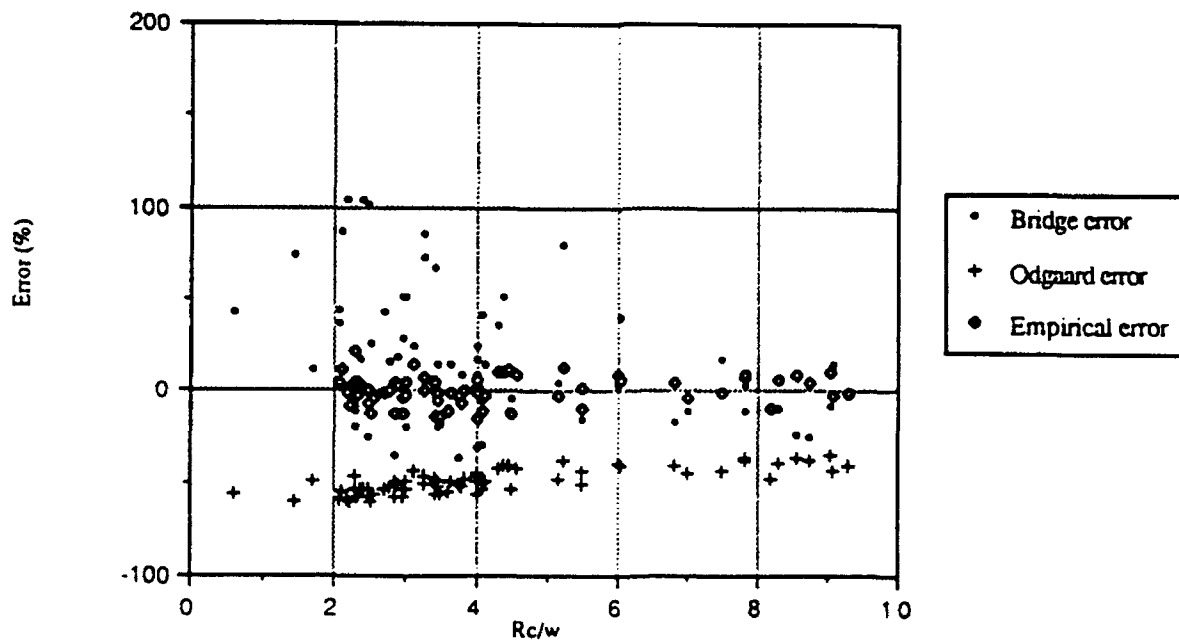


Fig. 15 Errors in Predicted Maximum Bend Scour Depths for Red River 1969 data
(Note empirical method was developed for this river)

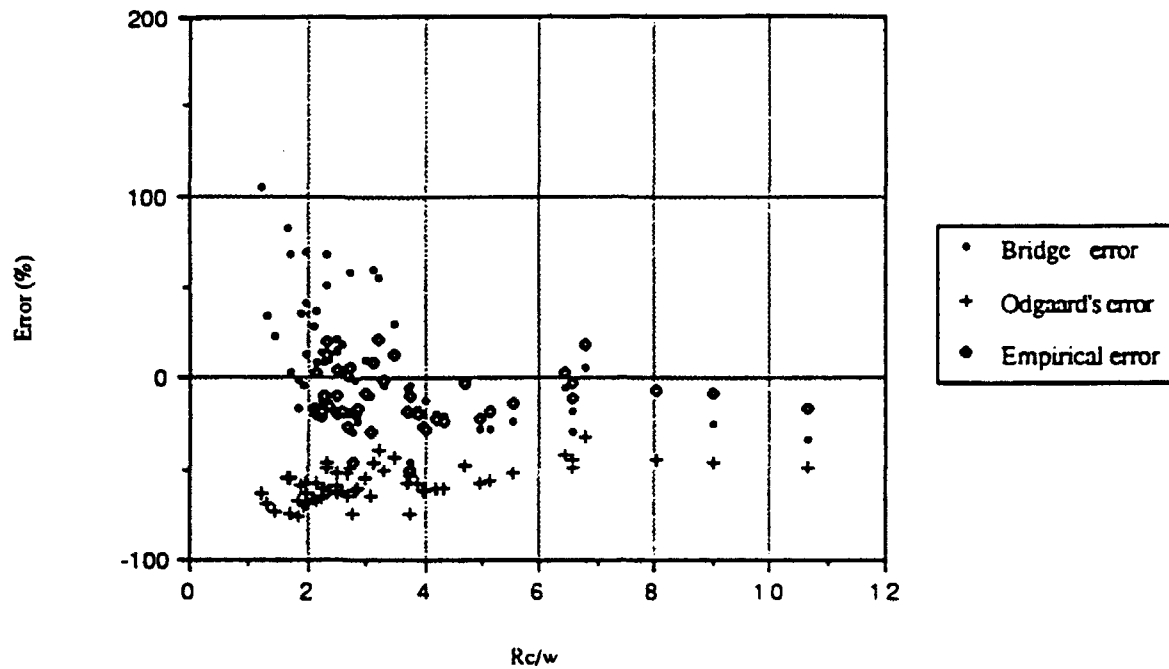


Fig. 16 Errors in Predicted Maximum Bend Scour Depth
for British Gravel-Bed Rivers

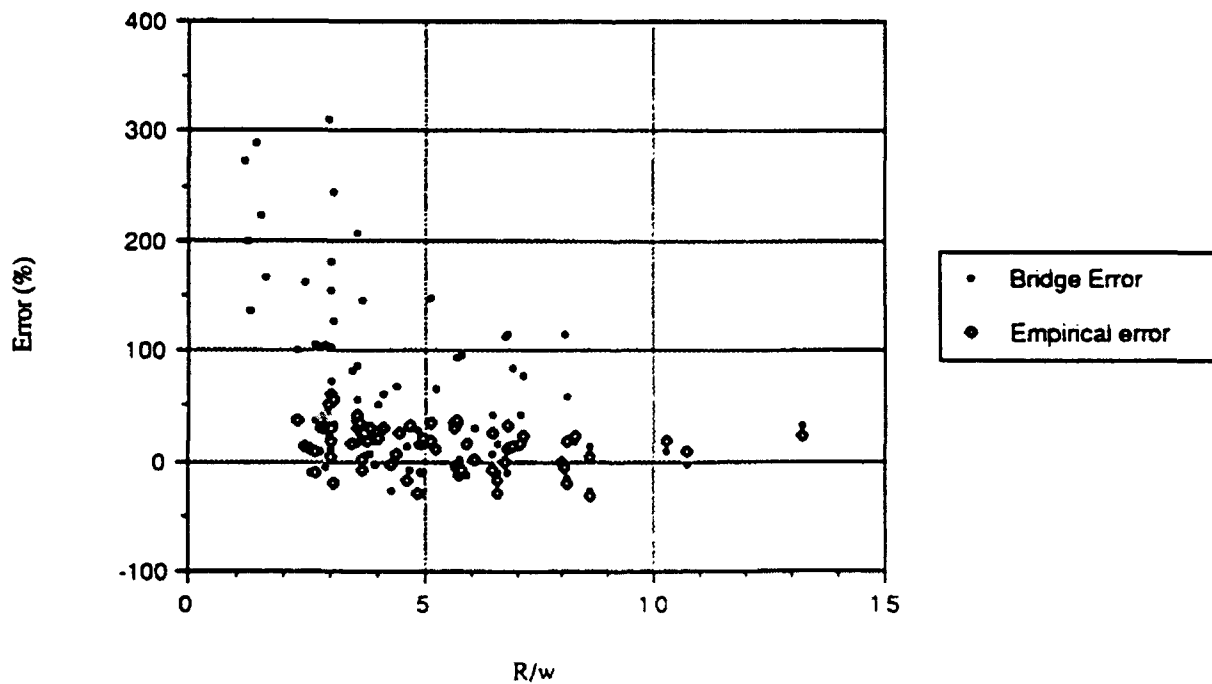


Fig. 17 Errors in Predicted Maximum Bend Scour Depths
for data from various researchers

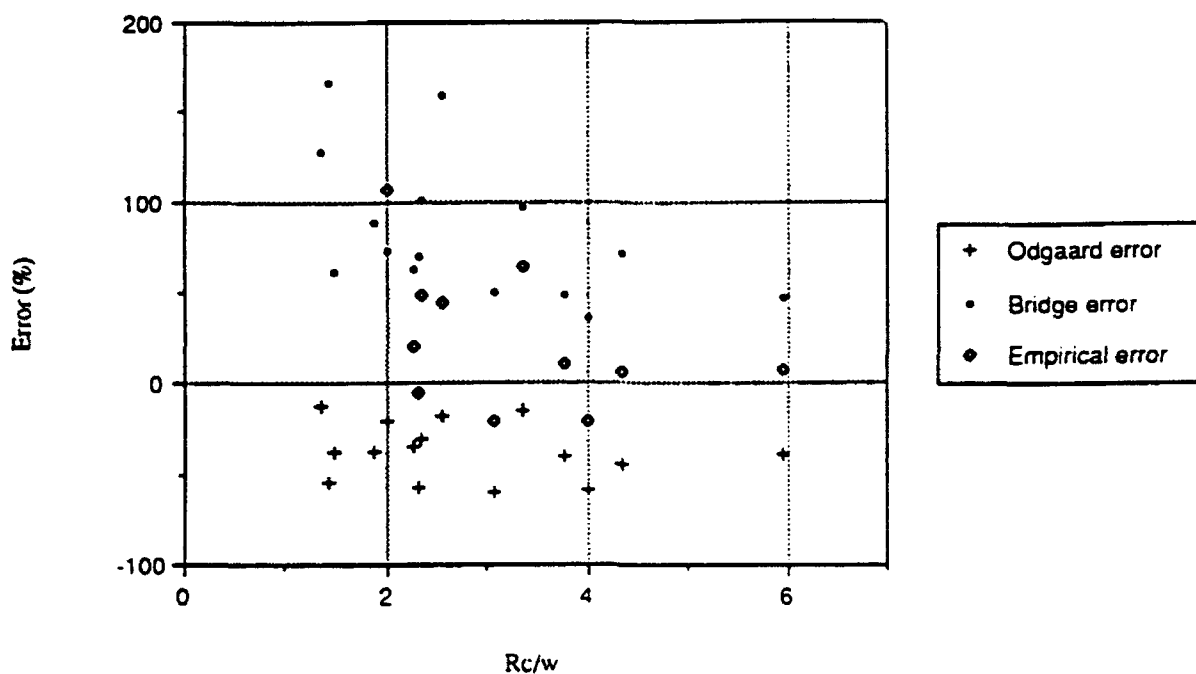


Fig. 18 Errors in Predicted Scour Depths
for selected Flume Data

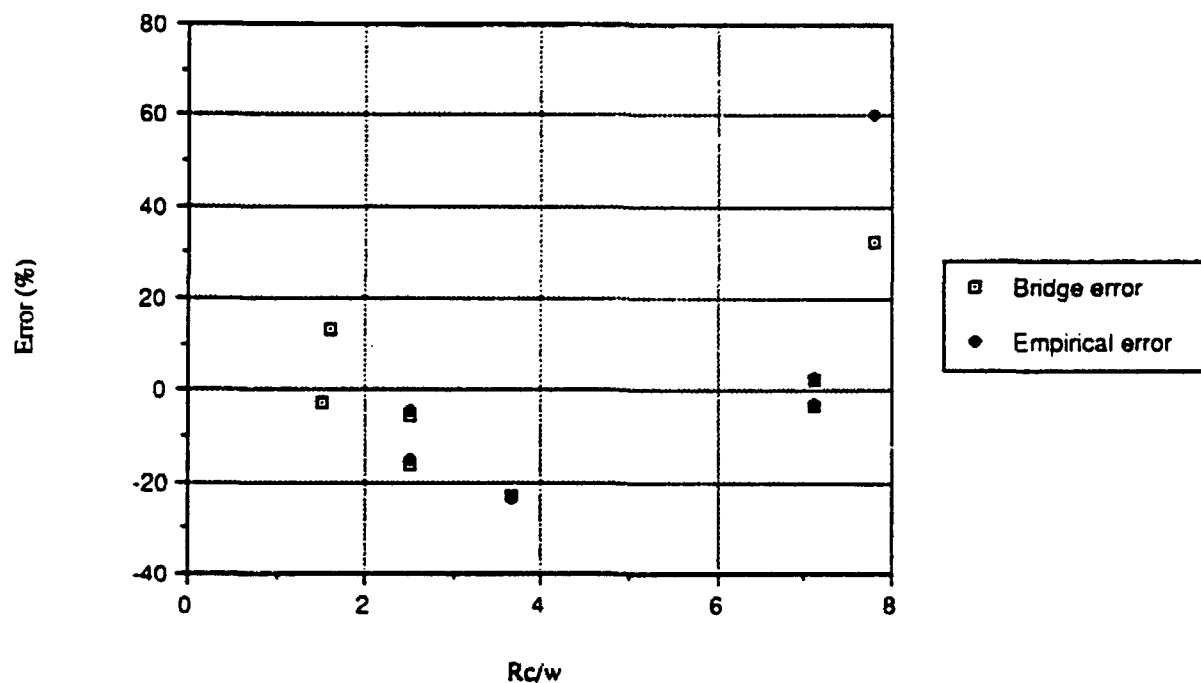


Fig. 19 Observed vs Predicted Maximum Bend Scour Depth for All Data

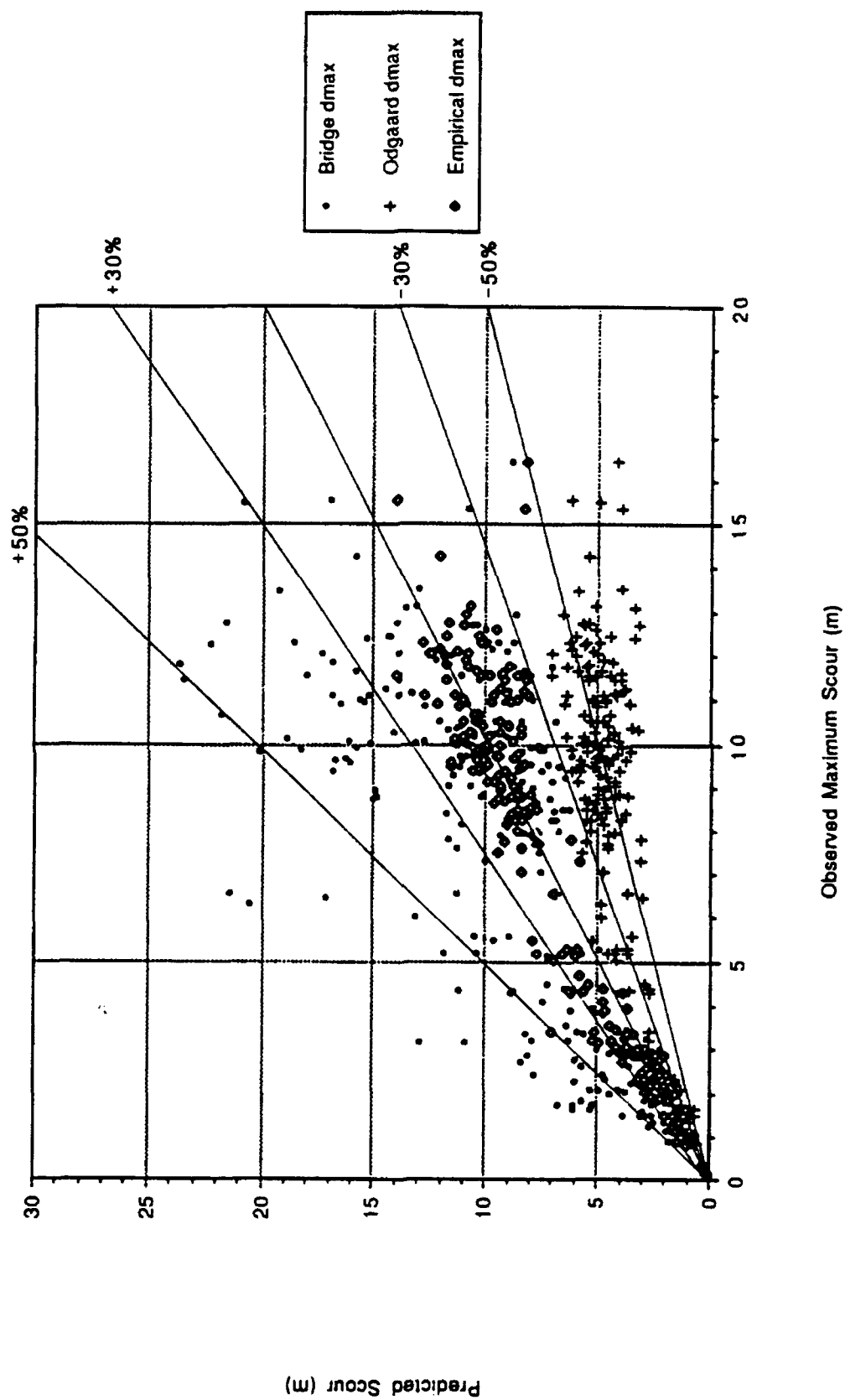


Fig. 20 Errors in Predicted Maximum Scour Depths for All Data

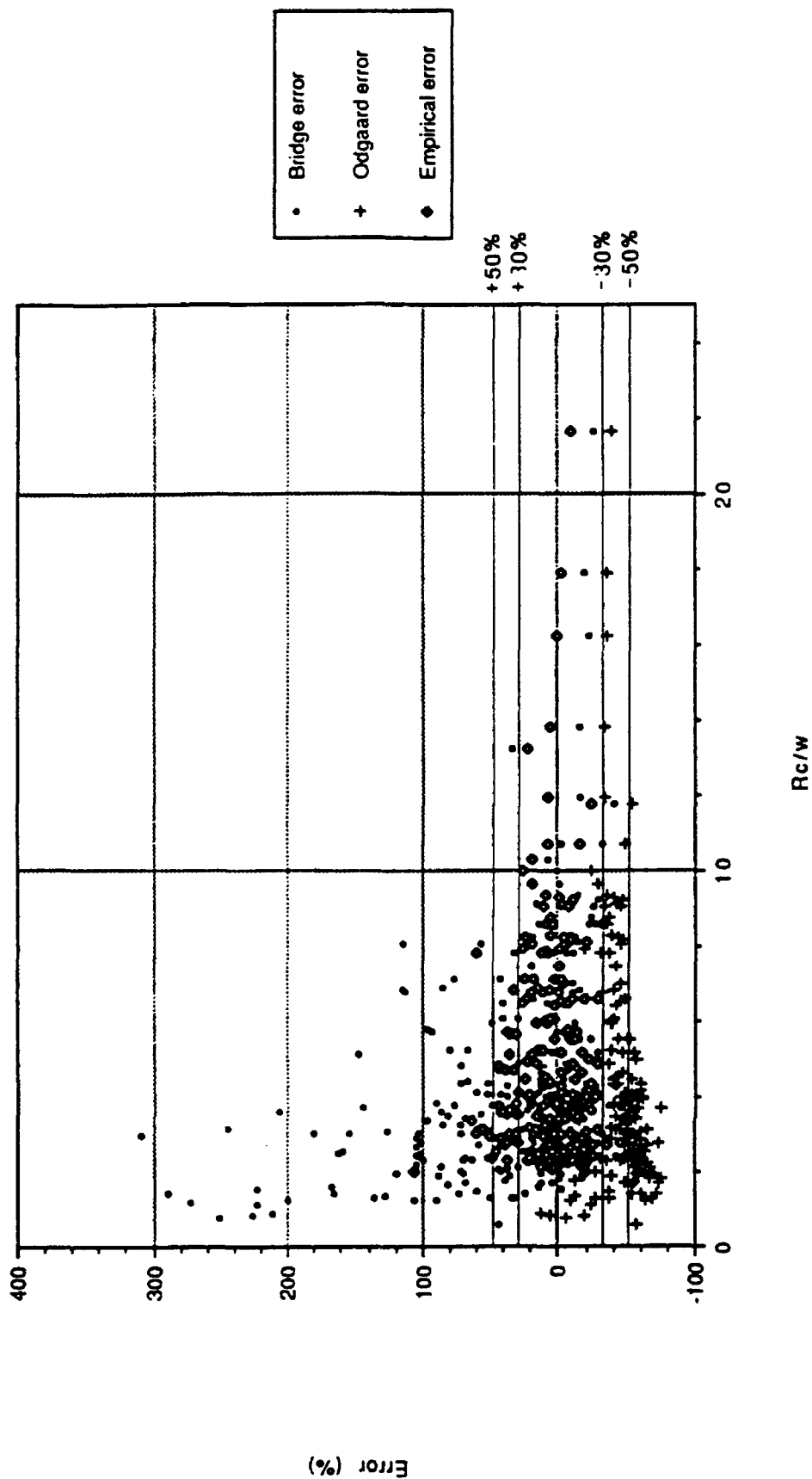


Fig. 21 Observed vs Predicted Maximum Scour for Revetted Bends

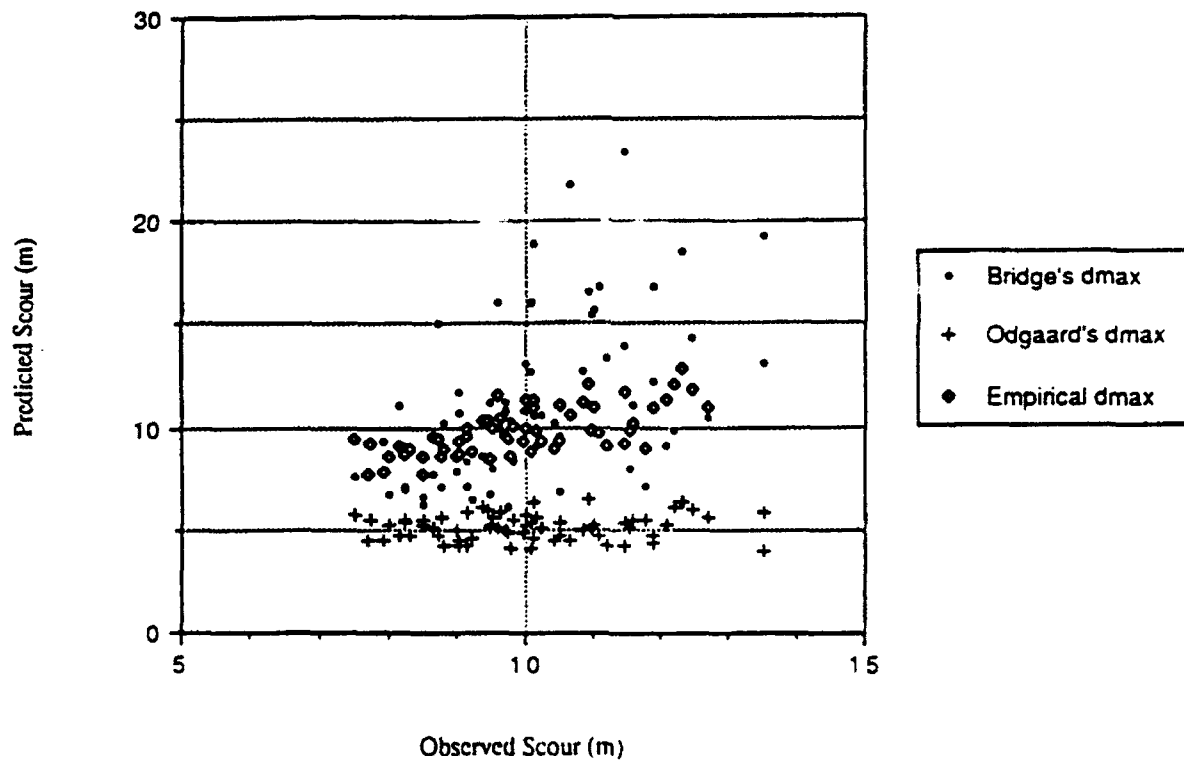


Fig. 22 Errors in Predicted Maximum Scour for Revetted Bends

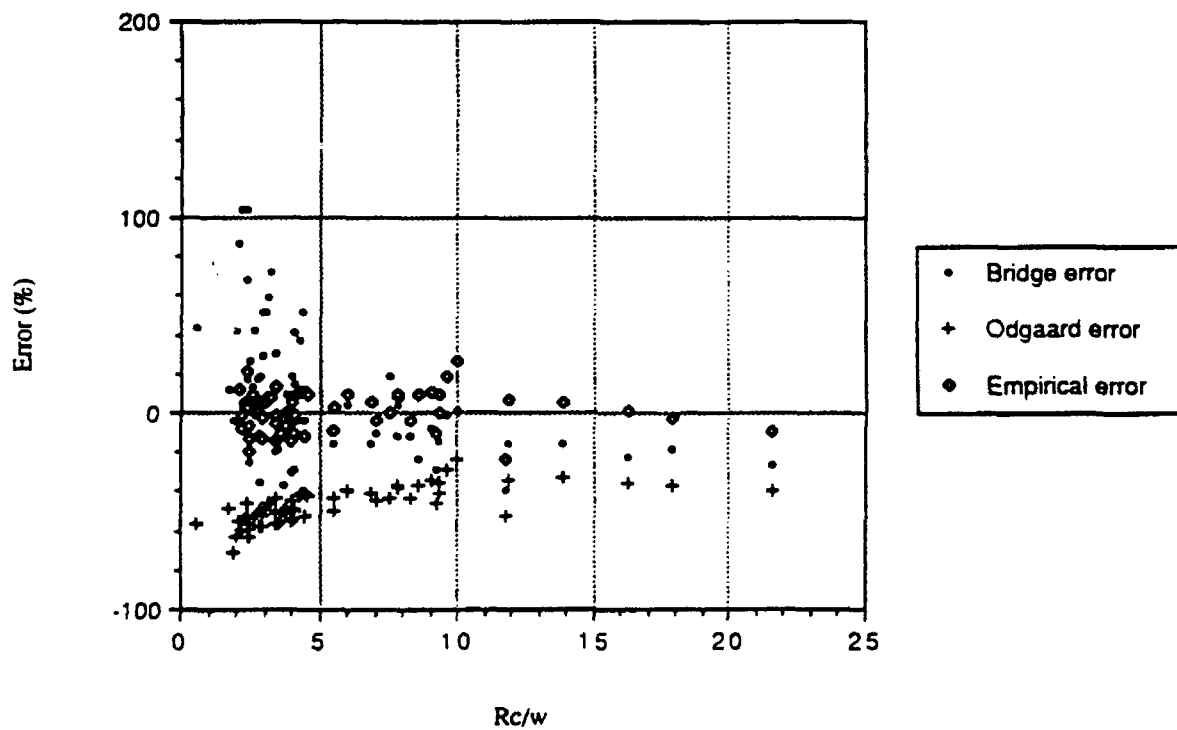


Fig. 23 Semi-log Plot of Dimensionless Maximum Bend Scour Depth for Reverted Bends

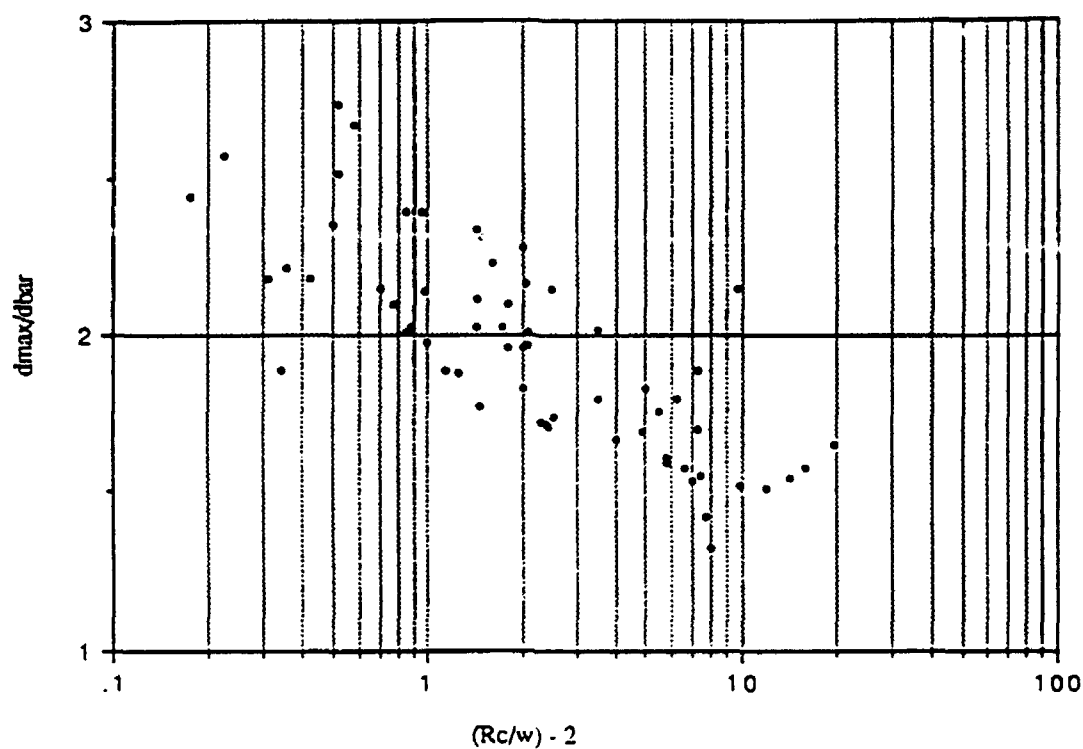


Fig. 24 Observed vs Predicted Maximum Scour Depths for Revetted Bends

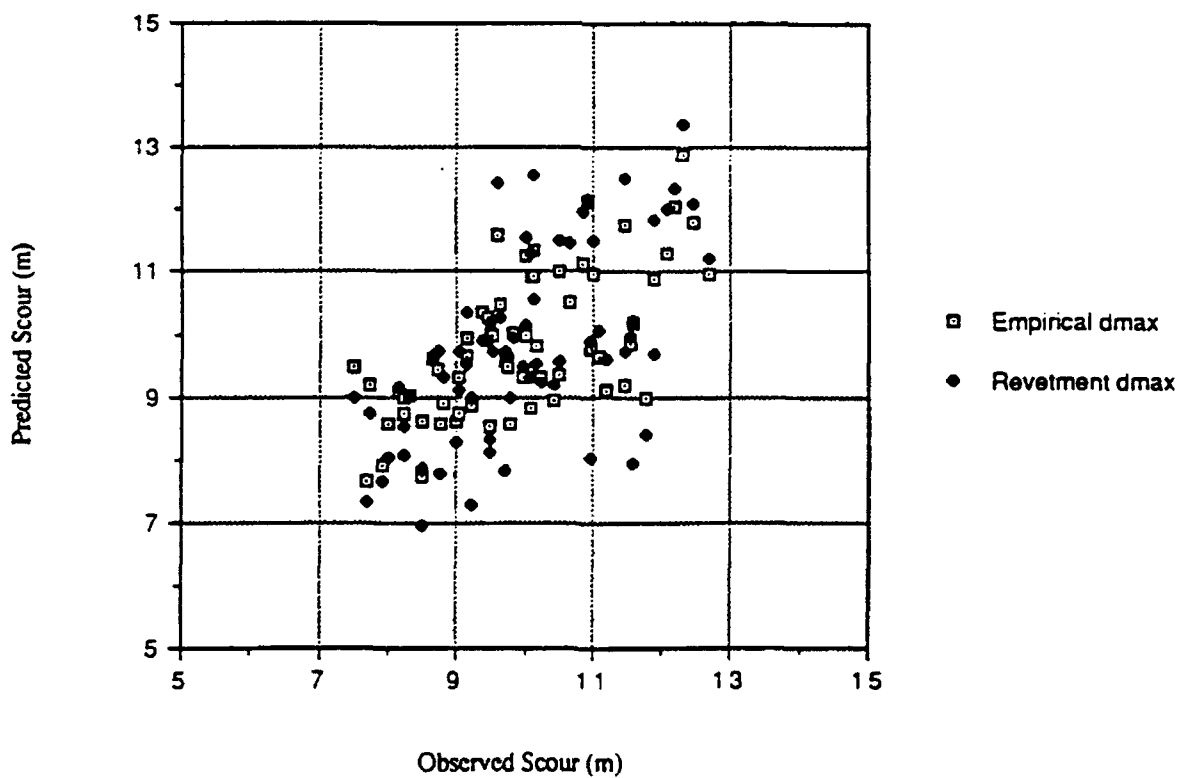


Fig. 25 Errors in Predicted Maximum Scour Depths for Reverted Bends

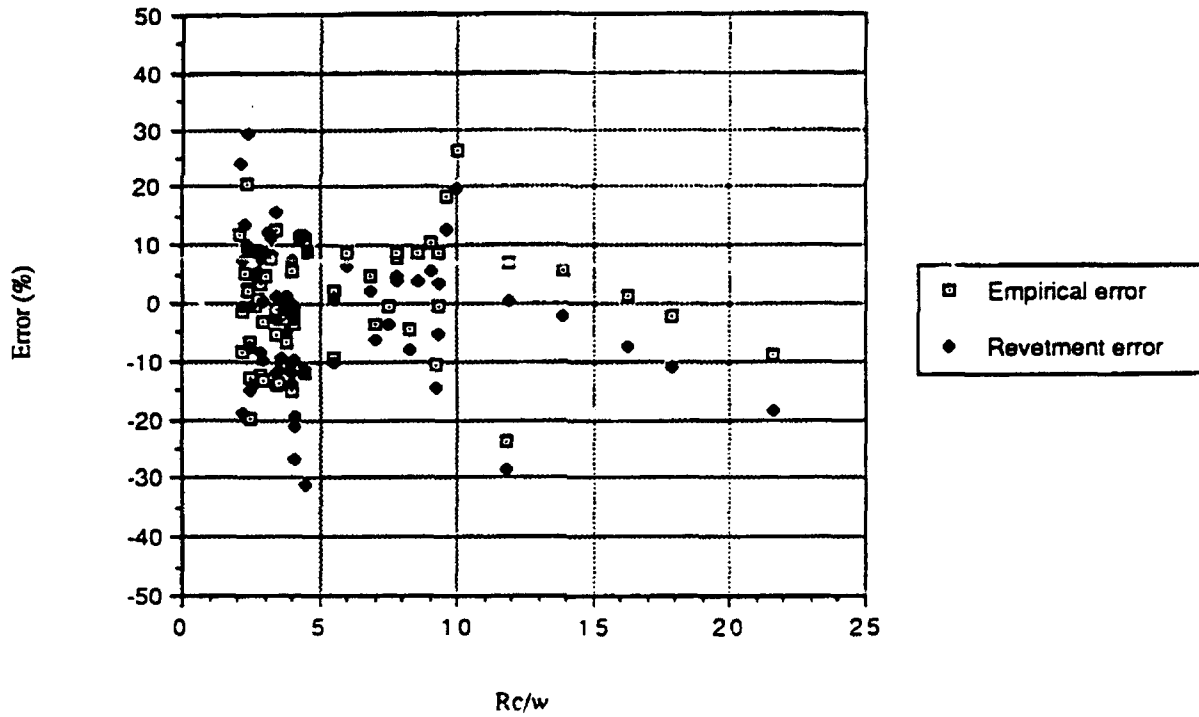


Fig. 26 Semilog Plot of Dimensionless Maximum Bend Scour Depth for Free Meanders

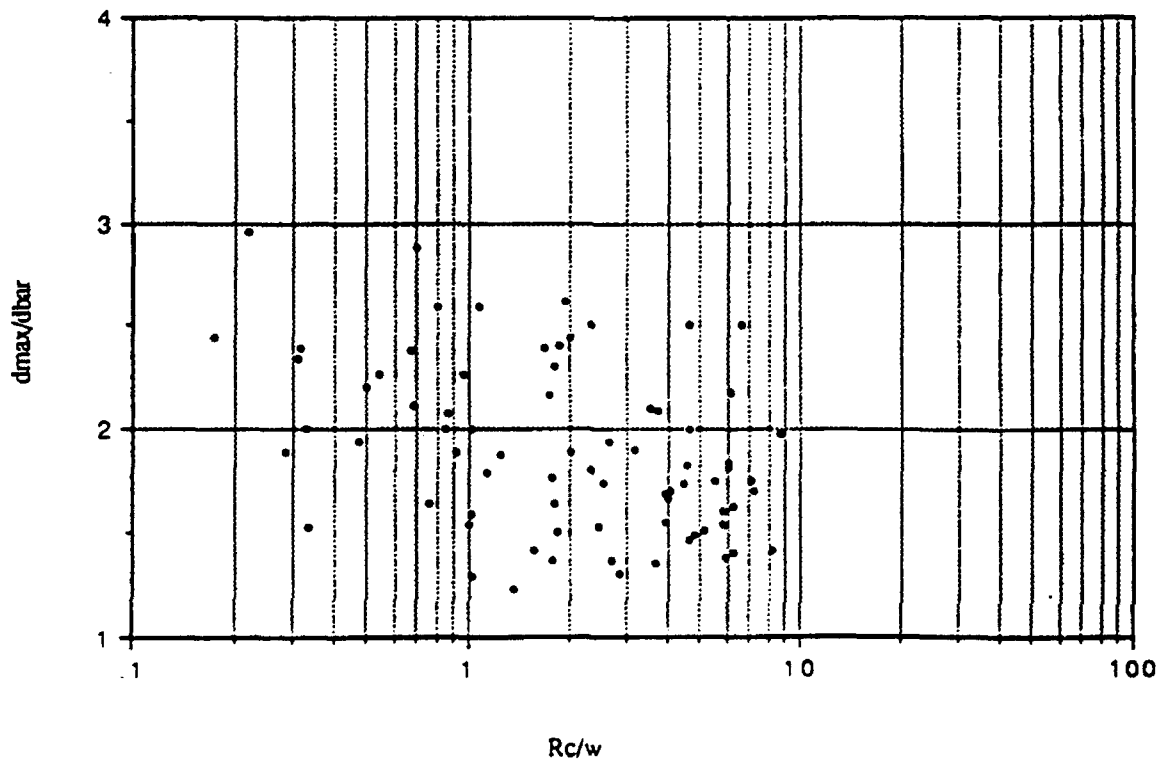


TABLE 1 - BASED DATA FOR NATURAL RIVERS FROM THORNE AND ABT (1990)

RESEARCHER	RIVER	SITE	BEND NUMBER	RADIUS OF CURVATURE (m)	MEANDER WAVELENGTH (m)	WIDTH (m)	MEAN DEPTH (m)	X-SECT SHAPE ...N/T/R...	OUTER BANK ANGLE (Degrees)
Markham/Thorne	Fall	Reach B	1	23.50	82.00	8.20	0.65	N	60
Thorne et al.	Fall	Reach A	1	10.30	38.00	12.50	0.76	N	75
Thorne et al.	Fall	Reach A	2	8.30	32.00	11.00	0.92	N	85
Thorne et al.	Fall	Reach A	3	8.71	42.00	9.90	0.96	N	70
Markham/Thorne	Roding	Loughon	1	21.00	132.00	12.00	1.30	N	60
C.R. Thorne	Fall	Reach 1	1	11.00	101.60	8.80	0.89	N	65
C.R. Thorne	Fall	Reach 1	2	13.50	27.00	10.60	0.66	N	64
D. Anthony	Fall	Reach 4	1	13.75	120.80	10.81	0.79	N	63
J.S. Bridge	South Esk	Glen Cova	1	67.10	230.00	23.00	1.22	N	58
N.G. Bhowmik	Kaskaski	Reach 1	1	301.80	664.46	38.10	3.77	N	34
N.G. Bhowmik	Kaskaski	Reach 1	2	298.70	573.20	45.40	3.59	N	45
N.G. Bhowmik	Kaskaski	Reach 1	3	136.60	408.40	36.30	4.01	N	30
N.G. Bhowmik	Kaskaski	Reach 1	4	40.40	207.46	36.30	3.84	N	46
N.G. Bhowmik	Kaskaski	Reach 1	5	32.00	170.90	39.90	3.48	N	58
N.G. Bhowmik	Kaskaski	Reach 2	2	380.40	1011.98	48.60	3.41	N	61
N.G. Bhowmik	Kaskaski	Reach 2	3	91.40	597.10	47.10	3.68	N	53
N.G. Bhowmik	Kaskaski	Reach 2	4	2.3.40	743.96	45.40	3.69	N	51
N.G. Bhowmik	Severn	Reach 2	1	95.00	82.00	25.00	0.65	N	80
Bahurst/Thorne	Severn	Mac's Mawr	1	44.00	65.00	9.10	0.87	N	90
S. Maynard	Missouri	Rickety Bridge	1	3625.50	8046.00	202.50	5.55	N	27
S. Maynard	Missouri	Browers Bend	2	3238.50	8690.00	199.50	5.50	N	22
S. Maynard	Missouri	Snyder Bend	3	2000.25	8046.50	200.00	5.65	N	23
S. Maynard	Missouri	Glovers Point Bend	4	1952.63	9655.80	209.00	5.30	N	28
S. Maynard	Missouri	Winnebago Bend	5	1857.38	3218.60	225.75	5.00	N	25
S. Maynard	Missouri	Upper Omaha Mission	6	2714.63	8047.00	196.00	5.45	N	22
S. Maynard	Missouri	Middle Omaha Mission	7	2047.88	4827.90	212.50	5.45	N	28
S. Maynard	Missouri	Lower Omaha Bend	8	2143.13	3218.60	231.50	5.03	N	36
S. Maynard	Missouri	Upper Monona Bend	9	2619.38	11265.10	223.00	5.48	N	26
S. Maynard	Missouri	Lower Monona Bend	10	4524.38	11265.10	209.25	5.15	N	21
S. Maynard	Missouri	Blackbird Bend	11	2381.25	8046.50	199.75	5.25	N	22
de Vriend/Geldorf	Dommel	Tieville Bend	1	16.00	95.20	5.88	0.50	N	74
de Vriend/Geldorf	Dommel	The Netherlands	2	14.50	63.40	6.00	0.50	N	64
Dietrich & Smith	Muddy Creek	Wyoming	1	8.00	50.00	4.00	0.40	N	67
A.J. Odgaard	E. Nishabouna	Iowa	1	233.00	1120.00	48.00	2.05	N	47

N = Natural

Explanation

TABLE 1 (Continued) - BASE DATA FOR NATURAL RIVERS FROM THORNE AND ABT (1990)

OUTERBANK ROUGHNESS ...S/I/R...	D50 (m)	BEDFORMS ...R:D:P...	APPROACH CHANNEL ...S/M/B...	AVERAGE VELOCITY (m/s)	DEPTH-AVE TOE VELOCITY (m/s)	SLOPE (x 10 ⁻³)	FRICTION FACTOR	DISCHARGE (cumecs)	SINUOSITY
R	0.0140	P	S	0.53	0.80	1.46	0.27	2.82	1.43
R	0.0097	P	M	0.57	0.80	1.16	0.21	5.4	2.5
R	0.0038	P	M	0.46	0.60	2.14	0.73	4.65	1.64
R	0.0130	P	S	0.71	1.10	1.29	0.19	6.75	1.9
R	0.0130	P	S	1.13	1.35	1.8	0.14	17.63	1.59
I	0.0010	R,D	S	0.51	0.80	1.73	2.38	4	2.2
I	0.0010	R,D	M	0.58	0.74	1.73	0.25	4	2.2
I	0.0042	D	S	0.48	0.70	1.32	0.36	4	2.1
I	0.0018	R,D	S	0.48	0.69	5	2.08	13	1.1
R	0.0022	D	S	0.84	1.05	0.203	0.085	120.6	1.07
R	0.0005	R,D	M	0.86	0.95	0.203	0.077	140.2	1.07
R	0.0009	D	M	0.84	1.03	0.203	0.0905	122.3	1.22
R	0.0034	D	M	0.62	0.80	0.203	0.159	86.4	2.59
R	0.0057	D	S	0.69	0.93	0.203	0.116	95.8	2.67
R	0.0052	D	S	0.61	0.83	0.117	0.084	101.1	1.08
R	0.0025	D	M	0.61	0.74	0.117	0.091	10.7	4.36
R	0.0048	D	S	0.61	0.70	0.117	0.091	102.2	1.23
R	0.0317	P	S	0.94	0.98	1.61	0.093	15.3	1.11
R	0.0630	P	S	1.35	1.60	1.3	0.049	10.7	1.06
I	0.0003	R,D	M	1.32	1.46	0.19	0.048	1483.5	1.13
I	0.0003	R,D	S	1.36	1.53	0.19	0.044	1492.3	1.07
I	0.0003	R,D	S	1.27	1.62	0.19	0.052	1435.4	1.21
I	0.0003	R,D	M	1.35	1.55	0.19	0.043	1495.4	1.27
I	0.0003	R,D	M	1.42	1.69	0.19	0.037	1602.8	1.06
I	0.0003	R,D	M	1.44	1.81	0.19	0.039	1538.2	1.14
I	0.0003	R,D	M	1.40	1.55	0.19	0.041	1621.4	1.08
I	0.0003	R,D	M	1.37	1.55	0.19	0.04	1595.3	1.02
I	0.0003	R,D	M	1.47	1.67	0.19	0.038	1796.4	1.24
I	0.0003	R,D	S	1.45	1.54	0.19	0.037	1562.6	1.15
I	0.0003	R,D	M	1.42	1.69	0.19	0.039	1489.1	1.13
I	0.0010	D	S	0.43	0.55	0.52	0.11	1.3	1.3
I	0.0010	D	S	0.42	0.61	0.52	0.116	1.26	1.05
S	0.0007	R,D	M	0.55	0.75	1.4	0.145	0.88	1.07
R	0.0005	R,D	S	1.25	1.60	6.8	0.7	123	1.65

R = Rough
 I = Intermediate
 S = Smooth
 R = Ripples
 D = Dunes
 P = Plane
 S = Straight
 M = Meandering
 B = Braided

TABLE 2 - CALCULATED DATA FOR NATURAL RIVERS FROM THORNE AND ABT (1990)

Rc/w	dmax/dbar	MEASURED dmax (m)	BRIDGE'S dmax (m)	ODGAARD'S dmax (m)	EMPIRICAL dmax (m)	BRIDGE'S ERROR (%)	ODGAARD'S ERROR (%)	EMPIRICAL ERROR (%)
2.87	2.08	1.35	1.99	0.833	1.36	47.41	-38.30	0.98
0.82	2.16	1.64	5.3522	1.3321		226.35	-18.77	
0.75	1.88	1.73	6.087	1.6354		251.85	-5.47	
0.88	1.90	1.82	5.6725	2.0502		211.68	12.65	
1.75	2.22	2.89	4.01	2.5383		38.75	-12.17	
1.25	1.79	1.59	3.01	1.4438		89.31	-9.19	
1.27	2.64	1.74	2.62	1.2743		50.57	-26.76	
1.27	2.59	2.05	2.7	1.2708		31.71	-38.01	
2.92	1.89	2.3	2.56	1.5269	2.55	11.30	-33.61	10.67
7.92	1.37	5.18	5.67	4.2234	6.53	9.46	-18.47	26.06
6.58	1.47	5.27	6.16	4.1647	6.39	16.89	-20.97	21.32
3.76	1.37	5.48	9.64	5.2184	7.87	75.91	-4.77	43.59
1.12	1.65	6.34	20.55	4.8469		224.13	-23.55	
0.80	1.88	6.55	21.3808	6.8718		226.42	4.91	
7.83	1.55	5.27	4.93	3.6566	5.92	-6.45	-30.61	12.27
1.94	1.63	6	13.15	4.8383		119.17	-19.36	
4.70	1.36	5.03	7.21	4.1945	6.94	43.34	-16.61	38.01
3.80	2.31	1.5	2.85	0.8993	1.27	90.00	-40.05	-15.14
4.84	1.30	1.13	1.94	1.2688	1.63	71.68	12.28	44.13
17.90	1.58	8.75	7.12	5.5786	8.57	-18.63	-36.24	-2.04
16.23	1.55	8.5	6.55	5.532	8.61	-22.94	-34.92	1.29
10.00	1.33	7.5	7.61	5.7044	9.46	1.47	-23.94	26.17
9.34	1.56	8.25	7.07	5.3473	8.96	-14.30	-35.18	8.65
8.23	1.80	9	7.9	5.0411	8.61	-12.22	-43.99	-4.31
13.85	1.51	8.25	6.99	5.488	8.72	-15.27	-33.48	5.71
9.64	1.42	7.75	7.66	5.4965	9.18	-1.16	-29.08	18.40
9.26	1.89	9.5	6.72	5.0652	8.52	-29.26	-46.68	-10.34
11.75	2.14	11.75	7.06	5.5147	8.97	-39.91	-53.07	-23.63
21.62	1.65	8.5	6.28	5.1692	7.75	-26.12	-39.19	-8.85
11.92	1.52	8	6.71	5.2897	8.58	-16.13	-33.88	7.23
2.72	3.30	1.65	1.04	0.6406	1.07	-36.97	-61.18	-35.39
2.42	3.00	1.5	0.865	0.6549	1.12	-42.33	-56.34	-25.46
2.00	2.13	0.85	0.69	0.6219		-18.82	-26.84	
4.85	2.10	4.3	4.05	2.726	3.84	-5.81	-36.60	-10.81

TABLE 3 - BASE DATA FOR RED RIVER HYDROGRAPHIC SURVEY, 1981 FROM THORNE, 1988

SITE (River Mile)	RADIUS OF CURVATURE (m)	WIDTH (m)	MEAN DEPTH (m)	AVERAGE VELOCITY (m/s)	SLOPE (x 10 ⁻³)	FRICTION FACTOR	DISCHARGE (cumecs)	MEANER WAVELENGTH (m)	SINUOSITY
422.5	1219.2	304.8	4.9	1.671	0.14397	0.19837	2495	5140	1.6
420	1066.8	360.6	4.65	1.488	0.14362	0.23671	2495	3460	1.86
416	1066.8	312.4	4.69	1.703	0.17786	0.22575	2495	3660	1.89
413	762	302.7	4.01	2.055	0.17278	0.12869	2495	3660	1.96
411	1524	295	4.18	2.023	0.17902	0.14344	2495	6280	1.51
407	762	330.1	4	1.890	0.23343	0.20529	2495	4040	1.1
406	1143	449	3.71	1.498	0.23268	0.30279	2495	4940	1.16
404.8	457.2	320	3.73	2.090	0.20586	0.13792	2495	2120	1.78
402	457.2	221	4.85	2.328	0.23187	0.16288	2495	1780	1.46
401	2133.6	304.8	5.56	1.671	0.18447	0.28824	2832	5520	1.12
398.5	838.2	358.1	4.45	1.777	0.177	0.19571	2832	4140	1.27
397	914.4	330.1	4.2	2.043	0.15094	0.11924	2832	4140	1.31
394.5	762	365.8	4.21	1.839	0.144	0.1407	2832	3220	1.44
392.5	914.4	320	4.09	2.164	0.14417	0.09884	2832	3840	1.32
391.5	914.4	281.9	4.9	2.050	0.20123	0.1841	2832	2840	2.26
389.5	914.4	240.2	5.1	2.312	0.18431	0.3803	2832	4220	1.29
387	1752.6	289.6	4.94	1.980	0.11283	0.11703	2832	3120	1.19
386	1219.2	325.5	4.82	1.805	0.15684	0.18208	2832	5420	1.02
384.5	1066.8	297.2	4.72	2.019	0.15625	0.14202	2832	3460	1.12
383.5	609.6	266.7	5	2.124	0.13932	0.12122	2832	3260	1.06
382	609.6	251.5	5.26	2.141	0.10522	0.09477	2832	1728	1.84
377.8	762	304.8	5.36	1.733	0.07939	0.04175	2832	2832	5.32
376	3048	335.3	5.04	1.676	0.07578	0.1067	2832	3264	1.08
374.5	2743.2	335.3	5.07	1.666	0.12483	0.17897	2832	4320	1.07
373.5	609.6	358.1	4.87	1.624	0.13131	0.19031	2832	2736	1.07
372	914.4	263	5.49	1.961	0.06471	0.07247	2832	3456	1.12
370.5	762	282	5.13	1.958	0.13228	0.13897	2832	2832	1.43
367.5	1066.8	282	4.76	2.110	0.24551	0.20604	2832	4656	1.49
365	1066.8	358.1	4.22	1.874	0.20948	0.19755	2832	4320	1.48
361.4	1371.6	320	4.74	1.867	0.15862	0.16926	2832	4176	1.29
360	609.6	259.1	4.92	2.222	0.1894	0.14817	2832	3312	1.42
359	1219.2	274.3	5.06	2.040	0.13774	0.13138	2832	5760	1.11
356	1219.2	297.2	4.95	1.925	0.13234	0.13874	2832	5088	1.51
354	1371.6	335.3	4.6	1.836	0.13258	0.12892	2832	4704	1.03
353.5	2743.2	365.8	4.52	1.713	0.13258	0.16031	2832	2448	1.04
352.5	914.4	396.2	4.2	1.702	0.18515	0.2107	2832	3984	1.08
351.2	762	350.5	4.38	1.845	0.18454	0.18641	2832	1680	1.49

TABLE 3 (Continued) - BASE DATA FOR RED RIVER HYDROGRAPHIC SURVEY, 1981 FROM THORNE, 1988

SITE (River Mile)	RADIUS OF CURVATURE (m)	WIDTH (m)	MEAN DEPTH (m)	AVERAGE VELOCITY (m/s)	SLOPE ($\times 10^{-3}$)	FRICTION FACTOR	DISCHARGE (cumecs)	MEANDER WAVELENGTH (m)	SINUOSITY
349	990.6	304.8	4.66	2.001	0.18511	0.16909	2842	3312	1.72
347.5	762	342.9	4.63	1.790	0.2119	0.24028	2842	2832	1.15
345	1524	335.3	4.76	1.781	0.15134	0.1783	2842	5712	1.21
342	990.6	342.9	4.46	1.858	0.12374	0.12542	2842	3936	1.23
340.5	670.6	320	4.51	1.969	0.1498	0.13673	2842	2208	1.6
339	990.6	289.6	4.47	2.217	0.13363	0.09537	2870	4512	1.41
337.7	2971.8	320	4.53	1.980	0.09247	0.08386	2870	4944	1.11
334.7	914.4	365.8	5.14	1.684	0.09019	0.12823	3167	4080	1.06
330.5	914.4	320	5.25	1.885	0.11261	0.13057	3167	3552	1.01
328.5	1371.6	304.8	5.38	1.931	0.11336	0.12832	3167	5904	1.47
326.5	1219.2	304.8	5.18	2.006	0.11339	0.11456	3167	4032	1.21
325	1676.4	304.8	5.09	2.041	0.11341	0.10872	3167	4416	1.22
323	1219.2	304.8	5.08	2.045	0.11321	0.10788	3167	5280	1.09
318.5	1828.8	304.8	5.79	1.800	0.15727	0.22066	3176	4896	1.12
315	1676.4	304.8	5.48	1.901	0.13569	0.1614	3176	7440	1.16
310	1676.4	411.5	5.06	1.525	0.13685	0.23358	3176	3744	1.33
309	304.8	510.5	5.78	1.076	0.182	0.71254	3176	3408	1.21
307	2438.4	312.4	6.31	1.611	0.18939	0.36128	3176	2784	1.03
306.7	2133.6	243.8	6.09	2.139	0.18437	0.19258	3176	3600	1.01
304	2286	335.2	5.63	1.682	0.12648	0.1974	3176	8496	1.11
302.4	1219.2	335.3	5.25	1.804	0.17633	0.22319	3176	5232	1.43
299	1828.8	330.5	5.74	1.579	0.0947	0.17119	3176	4704	1.93
294	2286	266.7	5.81	2.050	0.13376	0.14517	3176	9744	1.06
292.2	2590.8	312.4	5.34	1.904	0.14663	0.16954	3176	3888	1.03
290	1524	510.5	4.96	1.254	0.0961	0.23777	3176	6114	1.06
288	1219.2	304.8	4.95	2.105	0.09191	0.08058	3176	4896	1.34
287	762	243.8	6.22	2.094	0.09058	0.1008	3176	3456	1.29
285	2133.6	236.2	6.09	2.208	0.20715	0.2021	3176	4224	1.05
283.5	914.4	304.8	6.22	1.675	0.29422	0.51179	3176	3840	1.8
280.5	1066.8	243.8	6.36	2.048	0.26807	0.31892	3176	4080	2.05
278.5	838.2	243.8	5.9	2.402	0.2649	0.21261	3455	3456	1.42
277.5	1905	243.8	5.92	2.394	0.19538	0.15841	3455	3408	1.03
276.5	838.2	243.8	6.02	2.354	0.26554	0.22638	3455	2976	1.05

TABLE 4 - CALCULATED DATA FOR RED RIVER HYDROGRAPHIC SURVEY, 1981

Rc/w	dmax/dbar	MEASURED dmax (m)	BRIDGES dmax (m)	ODGAARD'S dmax (m)	EMPIRICAL dmax (m)	BRIDGES ERROR (%)	ODGAARD'S ERROR (%)	EMPIRICAL ERROR (%)
4.00	1.89	9.27	11.50	4.94	9.50	24.04	-46.73	2.46
2.96	2.39	11.10	16.86	4.68	9.66	51.92	-57.81	-12.95
3.41	1.91	8.95	15.00	4.73	9.40	67.60	-47.15	5.02
2.52	2.52	10.09	12.72	4.05	8.80	26.11	-59.83	-12.76
5.17	1.90	7.94	8.32	4.20	7.74	4.74	-47.05	-2.55
2.31	2.35	9.39	7.54	4.04	9.17	-19.71	-56.96	-2.30
2.55	2.27	8.41	8.34	3.73	8.11	-0.87	-55.68	-3.61
1.43	2.58	9.64	16.76	3.79		73.88	-60.68	
2.07	2.48	12.05	17.40	4.99	12.51	44.38	-58.56	3.79
7.00	1.83	10.17	9.07	5.59	9.81	-10.81	-45.06	-3.55
2.34	2.34	10.42	11.07	4.49	10.12	6.24	-56.91	-2.86
2.77	2.10	8.81	10.27	4.24	8.90	16.53	-51.93	1.05
2.08	2.44	10.29	14.10	4.25	10.70	37.03	-58.71	4.03
2.86	2.39	9.78	10.36	4.12	8.59	5.92	-57.83	-12.21
3.24	2.01	9.87	18.29	4.96	9.94	85.35	-49.78	0.71
3.81	1.96	10.00	10.08	5.17	9.98	0.84	-48.29	-0.16
6.05	1.70	8.39	11.74	4.97	8.91	39.98	-40.79	6.23
3.75	2.02	9.74	6.16	4.86	9.47	-36.75	-50.15	-2.80
3.59	2.23	10.53	9.54	4.76	9.35	-9.38	-54.78	-11.16
2.29	1.90	9.48	8.42	5.09	11.54	-11.16	-46.30	21.73
2.42	2.18	11.47	23.42	5.36	11.75	104.14	-53.24	2.41
2.50	2.20	11.79	23.66	5.43	11.80	100.72	-53.95	0.09
9.09	1.75	8.82	10.16	5.05	8.56	15.17	-42.70	-2.98
8.18	1.31	9.68	8.62	5.09	8.74	-11.00	-47.46	-9.71
1.70	1.99	9.71	10.88	4.93		12.05	-49.19	
3.48	2.31	12.69	10.41	5.56	10.96	-18.00	-56.19	-13.65
2.70	2.14	11.00	15.66	5.20	10.96	42.36	-52.71	-0.33
3.78	2.10	9.98	10.87	4.81	9.33	8.94	-51.83	-6.51
2.98	2.14	9.04	11.70	4.25	8.75	29.44	-52.99	-3.18
4.29	1.72	8.17	11.13	4.77	9.07	36.19	-41.61	10.98
2.35	2.21	10.87	12.75	5.01	11.16	17.32	-53.89	2.65
4.44	1.71	8.65	7.67	5.11	9.61	-11.34	-40.98	11.15
4.10	1.97	9.73	11.15	4.99	9.55	14.61	-48.72	-1.87
4.09	2.00	9.22	6.49	4.63	8.88	-29.56	-49.81	-3.72
7.50	1.75	7.92	9.35	4.53	7.89	18.07	-42.78	-0.35
2.31	2.18	9.14	8.36	4.23	9.63	-8.55	-53.72	5.40
2.17	2.44	10.68	21.81	4.42	10.52	104.19	-58.57	-1.48

TABLE 4 (Continued) - CALCULATED DATA FOR RED RIVER HYDROGRAPHIC SURVEY, 1981

Rc/w	dmax/dbar	MEASURED dmax (m)	BRIDGE'S dmax (m)	ODGAARD'S dmax (m)	EMPIRICAL dmax (m)	BRIDGE'S ERROR (%)	ODGAARD'S ERROR (%)	EMPIRICAL ERROR (%)
3.25	1.88	8.74	15.07	4.70	9.45	72.39	-46.18	8.11
2.22	2.57	11.90	12.22	4.68	10.91	2.65	-60.67	-8.34
4.55	1.74	8.29	8.94	4.79	9.01	7.82	-42.26	8.66
2.89	2.02	9.02	10.71	4.49	9.33	18.69	-50.17	3.46
2.10	2.25	10.13	18.87	4.57	11.35	86.26	-54.92	12.01
3.42	2.33	10.43	10.19	4.51	8.95	-2.31	-56.75	-14.14
9.29	1.70	7.70	7.79	4.54	7.67	1.15	-41.01	-0.42
2.50	2.35	12.09	9.03	5.18	11.32	-25.34	-57.12	-6.39
2.86	2.01	10.53	6.81	5.30	11.02	-35.31	-49.63	4.66
4.50	2.15	11.56	11.11	5.42	10.20	-3.93	-53.12	-11.76
4.00	1.83	9.48	11.17	5.22	10.04	17.78	-44.92	5.91
5.50	2.02	10.26	10.53	5.12	9.32	2.64	-50.11	-9.12
4.00	2.27	11.55	8.00	5.12	9.85	-30.72	-55.67	-14.75
6.00	1.66	9.63	9.96	5.82	10.46	3.38	-39.51	8.62
5.50	1.80	9.84	8.35	5.51	10.04	-15.13	-43.96	2.02
4.07	2.17	10.96	15.47	5.08	9.77	41.18	-53.64	-10.83
0.60	2.34	13.50	19.28	5.89	10.95	42.81	-56.41	7.91
7.81	1.61	10.15	10.55	6.34	10.40	3.97	-37.54	5.33
8.75	1.62	9.87	7.43	6.13	10.40	-24.70	-37.87	5.33
6.82	1.69	9.51	8.00	5.65	9.97	-15.86	-40.55	4.86
3.64	2.01	10.53	12.08	5.29	10.38	14.68	-49.76	-1.46
5.22	1.63	9.37	16.90	5.77	10.61	80.40	-38.44	13.20
8.57	1.58	9.16	7.05	5.84	9.95	-23.09	-36.22	8.61
8.29	1.63	8.71	7.84	5.36	9.19	-9.98	-38.45	5.48
2.99	2.12	10.52	8.41	4.98	10.28	-20.04	-52.69	-2.27
4.00	1.96	9.71	10.64	4.99	9.59	9.61	-48.64	-1.19
3.13	1.78	11.10	13.89	6.34	12.74	25.13	-42.91	14.74
9.03	1.54	9.36	8.61	6.13	10.35	-8.05	-34.47	10.57
3.00	1.98	12.30	18.60	6.30	12.88	51.19	-48.77	4.68
4.38	1.72	10.92	16.54	6.45	12.12	51.47	-40.91	10.99
3.44	2.11	12.46	14.32	6.00	11.81	14.96	-51.82	-5.25
7.81	1.59	9.44	8.37	5.96	10.27	-11.36	-36.82	8.84
3.44	2.02	12.17	9.78	6.13	12.05	-19.67	-49.66	-1.02

TABLE 5 - BASE DATA FOR RED RIVER HYDROGRAPHIC SURVEY, 1969

SITE (River Mile)	RADIUS OF CURVATURE (m)	WIDTH (m)	MEAN DEPTH (m)	AVERAGE VELOCITY (m/s)	SLOPE (x10-4)	FRICTION FACTOR	DISCHARGE (cumes)	MEANDER WAVELENGTH (m)	SINUOSITY
424.21	2590.8	322.463	4.653	1.663	1.45	0.063	2495		
423.07	990.6	330.082	3.819	1.979	1.44	0.036	2495		
422.06	838.2	406.279	3.144	1.953	1.45	0.031	2495		
420.16	990.6	386.163	5.055	1.278	1.47	0.117	2495		
416.34	762	431.881	4.012	1.440	1.78	0.088	2495		
413.31	685.8	406.279	4.342	1.414	1.72	0.096	2495	3190	1.52
410.74	1447.8	467.236	3.98	1.342	1.73	0.098	2495	6150	1.46
408.82	762	540.994	3.382	1.364	2.33	0.108	2495	3050	1.48
407.43	990.6	469.979	3.753	1.415	2.34	0.114	2495	3320	1.44
406	914.4	320.024	4.874	1.600	2.33	0.113	2495	3120	1.07
405.02	1143	264.249	5.02	1.881	2.33	0.085	2495	4100	1.22
402.09	457.2	279.488	5.483	1.628	2.34	0.125	2495	1960	1.54
400.39	2362.2	348.065	4.755	1.711	2.36	0.099	2832	2080	1.01
398.55	762	304.785	4.922	1.888	1.76	0.063	2832	2570	1.2
396.54	838.2	427.309	3.211	2.064	1.53	0.029	2832	3280	1.42
395.2	685.8	372.143	3.112	2.445	1.46	0.02	2832	3410	1.53
393.81	2209.8	337.702	4.63	1.811	1.43	0.052	2832	2350	1.01
392.72	914.4	429.137	3.409	1.936	1.45	0.034	2832	2710	1.18
391.24	838.2	308.747	4.962	1.849	1.77	0.066	2832	3220	1.58
389.29	1219.2	261.506	5.5	1.969	2.16	0.079	2832	3670	1.13
386.22	914.4	325.206	5.064	1.720	1.56	0.068	2832	3820	1.33
384.63	1371.6	266.382	5.424	1.960	1.57	0.057	2832	4500	1.1
383.34	609.6	261.81	5.008	2.160	1.49	0.041	2832	2330	1.3
382.14	609.6	324.291	4.759	1.835	1.07	0.039	2832	2500	1.3
378.7	762	231.027	5.858	2.093	0.616	0.021	2832	2450	1.09
377.57	685.8	214.569	6.877	1.919	0.668	0.032	2832	3080	1.63
376.04	2590.8	242.609	6.564	1.778	0.754	0.04	2832	3550	1.02
370.58	685.8	301.128	6.035	1.558	1.32	0.084	2832	2870	1.27
367.68	1066.8	424.261	4.186	1.595	2.45	0.104	2832	4800	1.98
365.28	762	386.163	4.278	1.714	2.09	0.078	2832	3010	1.65
361.58	1295.4	327.644	4.413	1.959	1.53	0.045	2832	5010	1.28
360.13	685.8	291.984	5.317	1.824	1.89	0.078	2832	2520	1.26
358.88	1371.6	213.35	6.937	1.914	1.53	0.074	2832	4760	1.2

TABLE 5 (Continued) - BASE DATA FOR RED RIVER HYDROGRAPHIC SURVEY, 1969

SITE (River Mile)	RADIUS OF CURVATURE (m)	WIDTH (m)	MEAN DEPTH (m)	AVERAGE VELOCITY (m/s)	SLOPE ($\pm 10^{-4}$)	FRICTION FACTOR	DISCHARGE (cumeecs)	MEANDER WAVELENGTH (m)	SINUOSITY
356.3	1066.8	413.898	4.188	1.634	1.32	0.053	2832	3840	1.41
354.26	1600.2	429.137	4.147	1.591	1.34	0.056	2832	6300	1.26
352.47	914.4	406.279	4.406	1.582	1.84	0.084	2832	2920	1.23
350.88	838.2	281.926	5.608	1.791	1.85	0.084	2832	3920	1.45
349.25	609.6	312.405	5.29	1.720	1.85	0.086	2842	2600	2.03
345.67	1219.2	314.843	4.555	1.982	1.68	0.051	2842	5300	1.28
342.33	990.6	586.711	3.379	1.434	1.24	0.053	2842	4440	1.65
335.92	1219.2	454.739	3.907	1.615	0.927	0.036	2870	3040	1.02
333.15	1905	515.696	4.272	1.438	1.03	0.067	3167	2980	1.04
331.63	2057.4	548.613	4.073	1.417	1.13	0.073	3167	4040	1.05
330.63	1219.2	548.613	3.926	1.470	1.14	0.065	3167	4210	1.12
328.66	1676.4	601.951	3.929	1.339	1.14	0.078	3167	6280	1.31
326.59	1219.2	487.656	4.581	1.418	1.13	0.082	3167	3850	1.1
324.89	2286	459.616	4.625	1.490	1.14	0.075	3167	5500	1.06
323.17	1600.2	594.331	3.883	1.372	1.13	0.072	3167	6200	1.16
321.34	1371.6	744.285	3.546	1.200	1.15	0.089	3167	3940	1.09
315.02	1676.4	416.641	4.162	1.832	0.758	0.024	3176	3140	1.08
310.21	990.6	462.359	4.671	1.471	1.38	0.078	3176	3860	1.34
308.87	533.4	411.46	4.848	1.592	1.67	0.082	3176	1950	1.24
306.77	2362.2	261.506	6.362	1.909	1.89	0.085	3176	3250	1.02
303.92	1981.2	474.855	3.977	1.682	1.27	0.046	3176	5960	1.14
302.33	914.4	477.598	3.968	1.676	1.89	0.07	3176	3380	1.23
299.58	685.8	563.852	3.558	1.583	0.947	0.035	3176	2500	1.75
294.15	1905	289.546	5.834	1.880	1.44	0.062	3176	6020	1.07
290.45	2590.8	467.236	4.517	1.505	1.08	0.056	3176	7730	1.09
288.06	914.4	391.039	5.087	1.597	0.926	0.048	3176	3340	1.32
286.69	914.4	264.249	5.648	2.128	0.915	0.03	3176	4010	1.41
283.36	914.4	291.984	5.346	2.035	2.67	0.09	3176	3660	1.81

TABLE 6 - CALCULATED DATA FOR RED RIVER HYDROGRAPHIC SURVEY, 1969

Rc/w	dmax/dbar	MEASURED dmax (m)	ODGAARD'S dmax (m)	BRIDGES dmax (m)	EMPIRICAL dmax (m)	BRIDGE'S ERROR (%)	ODGAARD'S ERROR (%)	EMPIRICAL ERROR (%)
8.03	1.84	8.549	4.6685		8.04		-45.39	-5.92
3.00			3.8464		7.90			
2.06			3.1625		8.16			
2.57			5.0929		11.01			
1.76			4.0427					
1.69	2.23	9.698	4.3834	16.2639		67.70	-54.80	
3.10	2.88	11.481	3.9946	10.3319	8.17	-10.01	-65.21	-28.86
1.41	3.66	12.383	3.4009	15.2959		23.52	-72.54	
2.11	3.00	11.256	3.7729	14.5088	9.36	28.90	-66.48	-16.87
2.86	2.53	12.31	4.9239	9.4925	10.23	-22.89	-60.00	-16.88
4.33	2.51	12.612	5.0714	10.0011	9.59	-20.70	-59.79	-23.99
1.64	2.23	12.231	5.6236	22.2413		81.84	-54.02	
6.79	1.49	7.086	4.772	7.5016	8.43	5.87	-32.66	18.94
2.50	2.44	12.027	4.9848	13.8765	10.84	15.38	-58.55	-9.90
1.96	3.22	10.335	3.2292	11.6428		12.65	-68.75	
1.84	4.08	12.71	3.1354	10.5802		-16.76	-75.33	
6.54	1.83	8.458	4.6472	6.8984	8.25	-18.44	-45.06	-2.43
2.13	3.04	10.378	3.4276	11.3035	8.37	8.92	-66.97	-19.31
2.71	2.00	9.948	5.0192	15.7835	10.59	58.66	-49.55	6.43
4.66	1.94	10.646	5.5571	10.2659	10.36	-3.57	-47.80	-2.67
2.81	2.60	13.145	5.115	13.0986	10.68	-0.35	-61.09	-18.73
5.15	2.27	12.295	5.4713	8.8023	10.05	-28.41	-55.50	-18.30
2.33	2.00	10.037	5.1009	15.1711	11.43	51.15	-49.18	13.84
1.88	2.45	11.661	4.8256	15.8586		36.00	-58.62	
3.30	2.05	12.024	5.9656	11.6075	11.84	-3.46	-50.39	-1.57
3.20	1.68	11.545	7.0533	18.0312	14.00	56.18	-38.91	21.28
10.68	1.97	12.953	6.6017	8.6571	10.89	-33.17	-49.03	-15.91
2.28	2.57	15.538	6.1339	17.0058	13.96	9.45	-60.52	-10.14
2.51	2.73	11.448	4.2108	13.9298	9.19	21.68	-63.22	-19.69
1.97	2.77	11.865	4.3171	16.8423		41.95	-63.61	
3.95	2.62	11.579	4.4407	9.3535	8.57	-19.22	-61.65	-25.96
2.35	2.68	14.273	5.4012	15.8399	12.07	10.98	-62.16	-15.43
6.43	1.74	12.066	7.0306	11.3761	12.40	-5.72	-41.73	2.75

TABLE 6 (Continued) - CALCULATED DATA FOR RED RIVER HYDROGRAPHIC SURVEY, 1969

Rc/w	dmax/dbar	MEASURED dmax (m)	ODGAARD'S d max (m)	BRIDGES d max (m)	EMPIRICAL dmax (m)	BRIDGES ERROR (%)	ODGAARD'S ERROR (%)	EMPIRICAL ERROR (%)
2.58	2.67	11.18	4.212	13.3258	9.11	19.19	-62.33	-18.55
3.73	3.97	16.443	4.1622	8.8482	8.15	-46.19	-74.69	-50.42
2.25	2.82	12.441	4.4427	14.3449	10.28	15.30	-64.29	-17.38
2.97	2.27	12.719	5.6864	13.9908	11.64	10.00	-55.29	-8.50
1.95	2.41	12.764	5.3773	21.6402		69.54	-57.87	
3.87	2.41	10.981	4.5878	9.1493	8.89	-16.68	-58.22	-19.08
1.69	3.88	13.094	3.3914	13.5534		3.51	-74.10	
2.68	2.11	8.251	3.9233	6.8034	8.37	-17.54	-52.45	1.47
3.69	2.39	10.222	4.2829	9.6225	8.42	-5.86	-58.10	-17.68
3.75	2.17	8.833	4.0818	8.4789	8.00	-4.01	-53.79	-9.45
2.22	2.95	11.597	3.9399	10.1932	9.25	-12.10	-66.03	-20.25
2.78	3.91	15.352	3.9382	10.8072	8.31	-29.60	-74.35	-45.85
2.50	2.11	9.653	4.6015	11.09	10.09	14.89	-52.33	4.48
4.97	2.38	11.003	4.6369	7.8643	8.62	-28.53	-57.86	-21.69
2.69	2.88	11.195	3.8926	9.0024	8.31	-19.59	-65.23	-25.78
1.84	3.08	10.911	3.5536	10.7822		-1.18	-67.43	
4.02	2.67	11.131	4.1763	9.7336	8.06	-12.55	-62.48	-27.61
2.14	2.38	11.106	4.6991	15.2362	11.40	37.19	-57.69	2.63
1.30	3.20	15.504	4.9119	20.8473		34.46	-68.32	
9.03	1.85	11.759	6.4	8.7757	10.81	-25.37	-45.57	-8.06
4.17	2.49	9.902	3.9872	8.0098	7.65	-19.11	-59.73	-22.78
1.91	3.40	13.511	3.9908	13.0518		-3.40	-70.46	
1.22	2.76	9.814	3.5778	20.1893		105.72	-63.54	
6.58	2.00	11.661	5.8696	8.2655	10.39	-29.12	-49.66	-10.90
5.54	2.11	9.525	4.5268	7.2483	8.26	-23.90	-52.47	-13.24
2.34	1.89	9.595	5.128	16.0979	11.58	67.77	-46.56	20.66
3.46	1.77	10.003	5.7224	13.0794	11.28	30.75	-42.79	12.82
3.13	1.89	10.079	5.4117	16.0734	10.94	59.47	-46.31	8.55

TABLE 7 - BASE DATA FOR BRITISH GRAVEL-BED RIVERS FROM HIEY AND THORNE (1986)

RIVER	RADIUS OF CURVATURE (m)	MEANDER WAVELENGTH (m)	WIDTH (m)	MEAN DEPTH (m)	VELOCITY (m/s)	SLOPE	FRICTION FACTOR	DISCHARGE (cumecs)	SINUOSITY
Otter	85.89	241.3	24.2	1.53	2.81	0.00353	0.054	104	1.13
Etc	167.22	241.3	24.7	1.57	2.68	0.00353	0.061	104	2.2
	97.55	179	32.4	1.64	2.90	0.00316	0.048	154	1.73
Camel	70.48	179	44.2	1.26	2.77	0.00316	0.041	154	1.25
	55.61	149.6	24.3	1.38	1.88	0.00420	0.129	63	1.18
Fowey	105.16	256.8	22.6	1.26	2.39	0.00226	0.039	68	1.3
West Dart	83.03	255.9	19.3	1.53	2.37	0.01271	0.272	70	1.03
	83.03	255.9	32.4	1.5	1.44	0.01271	0.721	70	1.03
Teign	163.16	359.7	31.1	2.01	2.37	0.00140	0.039	148	1.44
	175.62	359.7	24.7	2.5	2.40	0.00140	0.048	148	1.55
Erme	42.54	131.1	15.3	1.86	2.67	0.00640	0.131	76.1	1.03
Manifold A	42.34	64	13.8	0.76	2.67	0.00367	0.031	28	2.1
	46.37	64	15.8	2.07	0.86	0.00367	0.814	28	2.3
Manifold B	61.42	189.1	10.9	1.55	1.66	0.00189	0.084	28	1.03
	61.42	189.3	12.4	1.62	1.39	0.00189	0.124	28	1.03
Hamps	67.49	133.9	13.9	1.08	1.80	0.00482	0.126	27	1.6
	84.36	133.9	14.6	1.05	1.76	0.00482	0.128	27	2
Neath	91.33	281.5	30.9	2.46	2.26	0.00167	0.063	172	1.03
Usk	141.52	374.4	49.1	2.93	2.11	0.00133	0.068	304	1.2
	153.32	374.4	44	2.39	2.89	0.00133	0.030	304	1.3
Yscir	74.96	198.3	18.6	1.81	1.34	0.00300	0.239	45	1.2
Ceallog	72.43	221.1	18.3	1.35	1.94	0.00820	0.230	48	1.04
	72.43	221.1	14.8	1.17	2.77	0.00820	0.098	48	1.04
Lugg	27.19	66.4	21.3	0.85	1.33	0.00400	0.152	24	1.3
	24.05	66.4	20.2	1.28	0.93	0.00400	0.466	24	1.15
Lugg	45.76	80.7	15.2	1.03	1.53	0.00344	0.118	24	1.8
	28.98	80.7	23.7	1.14	0.89	0.00344	0.390	24	1.14
Pinsley Brook	13.74	24.36	9.7	0.94	1.54	0.00392	0.123	14	1.79
Dove	43.77	133.6	14.5	0.51	0.96	0.00611	0.265	7.1	1.04
	43.77	133.6	11.4	0.66	0.94	0.00611	0.355	7.1	1.04
Burbridge Brook	20.72	64.5	7.1	0.69	2.04	0.02147	0.279	10	1.02
	20.72	64.5	4.4	0.81	2.81	0.02147	0.173	10	1.02

TABLE 7 (Continued) - BASE DATA FOR BRITISH GRAVEL-BED RIVERS FROM HIEY AND THORNE (1986)

RIVER	RADIUS OF CURVATURE (m)	MEANDER WAVELENGTH (m)	WIDTH (m)	MEAN DEPTH (m)	VELOCITY (m/s)	SLOPE	FRICTION FACTOR	DISCHARGE (cumecs)	SINUOSITY
Churnet	56.18	127.4	12.8	1.65	1.61	0.00134	0.067	34	1.4
Rye	66.22	127.4	12.9	2.34	1.13	0.00134	0.194	34	1.65
	86.53	243.1	32.5	1.47	2.09	0.00356	0.094	100	1.13
	95.72	243.1	26.1	1.85	2.07	0.00356	0.121	100	1.25
	77.63	129.7	21	1.71	2.78	0.00278	0.048	100	1.9
Snaizholme Beck A	66.19	129.7	21.4	2.39	1.96	0.00278	0.136	100	1.62
	84.28	222.95	8.2	0.8	1.14	0.00525	0.252	7.5	1.2
	84.28	222.95	10.2	0.7	1.05	0.00525	0.262	7.5	1.2
Snaizholme Beck B	46.73	129	15.5	0.68	0.71	0.00343	0.361	7.5	1.15
	46.73	129	15.6	0.97	0.50	0.00343	1.063	7.5	1.15
	27.65	66	18.1	1.47	0.71	0.00241	0.345	19	1.33
Aster	63.36	156	10.8	1.33	1.32	0.00241	0.144	19	1.33
Frome	60.48	120	22.6	0.63	1.40	0.00356	0.089	20	1.6
	60.48	120	24.5	0.76	1.07	0.00356	0.184	20	1.6
Tweed (Hawick)	376.74	1150	63.7	2.08	2.70	0.00163	0.036	358.3	1.04
	383.99	1150	100.6	2.1	1.70	0.00163	0.094	358.3	1.06
Tweed (Boltonside)	235.94	700	34.8	1.59	1.65	0.00191	0.088	91.3	1.07
	197.77	576	30.5	1.62	1.85	0.00191	0.071	91.3	1.09
Tweed (Peebles)	179.55	500	31.6	2.26	2.15	0.00146	0.056	153.3	1.14
	71.06	188	19.9	1.44	1.74	0.00553	0.205	50	1.2
Sprint	71.06	188	17.2	1.23	2.36	0.00553	0.096	50	1.2
Rude	287.40	686	33.4	1.4	2.03	0.00382	0.102	95	1.33
	93.49	280	26.3	1.91	1.80	0.00333	0.154	90.5	1.06
Tarset Burn	93.49	280	25.7	2.1	1.68	0.00333	0.195	90.5	1.06
North Tyne	232.50	610	45.4	1.96	2.16	0.00251	0.083	192.3	1.21
	228.44	280	28.4	1.45	1.47	0.00119	0.063	60.4	2.59
Irthing	197.51	550	35.1	1.44	1.19	0.00119	0.094	60.4	1.14
Kielder Burn	111.13	210	16.9	0.89	2.43	0.00570	0.068	36.5	1.68
	119.60	226	33.4	1.93	0.57	0.00570	2.694	36.5	1.68
Mini	196.56	520	18.4	1.41	2.88	0.00736	0.098	74.7	1.2
	119.70	250	20.9	1.25	2.33	0.00273	0.049	61	1.52
Esk	108.11	176	19	1.24	2.59	0.00264	0.038	61	1.95

TABLE 7 (Continued) - BASE DATA FOR BRITISH GRAVEL-BED RIVERS FROM HEY AND THORNE (1986)

RIVER	RADIUS OF CURVATURE (m)	MEANDER WAVELENGTH (m)	WIDTH (m)	MEAN DEPTH (m)	VELOCITY (m/s)	SLOPE	FRICTION FACTOR	DISCHARGE (cumecs)	SINUOSITY
Hindburn	225.23	550	27.8	1.11	3.89	0.00670	0.039	120	1.3
Hodder	207.11	526	36.2	1.23	2.70	0.00670	0.089	120	1.25
	598.50	1250	45.2	2.92	2.64	0.00296	0.098	348	1.52
Eden	454.86	950	56.3	2.69	2.30	0.00296	0.118	348	1.52
	406.98	950	57.3	3.4	2.18	0.00170	0.096	424	1.36
Eden	205.46	526	74.2	2.65	1.21	0.00146	0.208	237	1.24
	235.66	674	52.7	3.06	1.47	0.00146	0.162	237	1.11
Glendaramackin	94.50	240	14.4	1.27	2.46	0.00469	0.077	45	1.25
	110.07	224	17	1.12	2.36	0.00469	0.074	45	1.56
Wylie	66.94	125	9.7	0.87	0.84	0.00157	0.152	7.1	1.7
	66.94	125	9.8	0.93	0.78	0.00157	0.189	7.1	1.7
Alwin	92.14	225	10.7	0.42	2.16	0.01086	0.077	9.7	1.3
	116.24	300	14.6	1.04	3.49	0.00927	0.062	53	1.23

TABLE 8-CALCULATED DATA FOR BRITISH GRAVEL-BED RIVERS FROM HEY AND THORNE (1986)

Rc/w	dmax/dbar	MEASURED dmax (m)	BRIDGE'S dmax (m)	EMPIRICAL dmax (m)	BRIDGE'S ERROR (%)	EMPIRICAL ERROR (%)
3.55	1.44	2.21	3.45	3.04	56.15	37.55
6.77	1.78	2.79	5.94	2.78	112.79	-0.22
3.01	1.76	2.88	8.09	3.39	180.82	17.76
1.59	1.79	2.25	6.00		166.64	
2.29	1.69	2.33	4.65	3.18	99.72	36.60
4.65	2.29	2.88	3.25	2.37	12.94	-17.55
4.30	1.96	3	2.20	2.92	-26.69	-2.51
2.56	1.96	2.94	2.65	3.27	-9.85	11.19
5.25	1.68	3.38	5.62	3.71	66.24	9.79
7.11	1.43	3.57	6.32	4.40	77.01	23.25
2.78	1.55	2.89	3.17	3.94	9.53	36.26
3.07	2.58	1.96	4.42	1.56	125.36	-20.22
2.93	1.53	3.158	12.93	4.31	309.55	36.53
5.63	1.40	2.17	2.06	2.83	-5.14	30.34
4.95	1.54	2.49	2.23	3.02	-10.46	21.29
4.86	2.64	2.85	3.65	2.02	27.95	-29.11
5.78	1.95	2.05	4.02	1.91	96.15	-6.91
2.96	1.39	3.41	4.08	5.11	19.50	49.95
2.88	1.47	4.3	8.83	6.13	105.30	42.67
3.48	1.72	4.12	7.46	4.77	81.05	15.73
4.03	1.60	2.89	4.39	3.50	51.87	21.22
3.96	1.61	2.18	2.11	2.62	-3.28	20.28
4.89	1.62	1.89	1.69	2.19	-10.43	15.64
1.28	2.45	2.08	4.91		136.22	
1.19	1.27	1.63	6.08		272.83	
3.01	2.01	2.07	5.26	2.13	154.31	2.90
1.22	1.53	1.74	5.21		199.57	
1.42	1.83	1.72	6.71		289.90	
3.02	1.59	0.81	0.89	1.05	10.47	30.11
3.84	1.50	0.99	1.04	1.29	5.38	30.28
2.92	1.61	1.11	1.06	1.44	-4.81	29.68
4.71	1.43	1.16	1.07	1.52	-7.95	31.32

TABLE 8 (Continued) - CALCULATED DATA FOR BRITISH GRAVEL-BED RIVERS FROM HEY AND THORNE (1986)

Rc/w	dmax/dbar	MEASURED dmax (m)	BRIDGES dmax (m)	EMPIRICAL dmax (m)	BRIDGES ERROR (%)	EMPIRICAL ERROR (%)
4.39	1.81	2.99	4.98	3.14	66.61	5.10
5.13	1.37	3.2	7.90	4.34	146.87	35.50
2.66	1.96	2.88	3.97	3.16	37.86	9.65
3.67	2.14	3.95	5.19	3.65	31.32	-7.60
3.70	1.96	3.35	8.18	3.37	144.30	0.54
3.09	1.32	3.15	10.88	4.91	245.26	55.78
10.28	1.41	1.13	1.21	1.33	7.36	18.12
8.26	1.40	0.98	1.16	1.21	18.16	22.96
3.01	1.29	0.88	1.79	1.41	102.86	59.74
3.00	1.54	1.49	2.56	2.01	71.62	34.81
1.53	1.65	2.43	7.85		223.16	
6.05	1.77	2.35	3.02	2.40	28.71	2.11
2.68	2.38	1.5	3.07	1.35	104.92	-9.94
2.47	1.95	1.48	3.88	1.68	161.84	13.69
5.91	1.56	3.24	2.84	3.77	-12.38	16.24
3.82	1.64	3.45	3.65	4.11	5.68	19.09
6.78	1.61	2.56	2.28	2.82	-11.07	10.11
6.48	1.43	2.31	2.47	2.89	6.87	25.17
5.68	1.32	2.99	4.04	4.12	35.13	37.74
3.57	1.40	2.02	3.77	2.86	86.64	41.45
4.13	1.50	1.84	2.94	2.37	59.61	28.76
8.60	1.64	2.3	2.62	2.40	13.81	4.17
3.55	1.54	2.95	3.43	3.79	16.25	28.59
3.64	1.46	3.07	3.73	4.15	21.47	35.18
5.12	1.56	3.06	4.19	3.63	36.82	18.74
8.04	1.82	2.64	5.66	2.51	114.44	-5.08
5.63	1.35	1.94	2.59	2.63	33.33	35.48
6.58	2.51	2.23	2.60	1.59	16.74	-28.92
3.58	1.41	2.72	8.36	3.83	207.49	40.70
10.68	1.55	2.18	2.11	2.34	-3.28	7.32
5.73	2.11	2.64	3.55	2.28	34.36	-13.82
5.69	1.95	2.42	4.68	2.26	93.35	-6.64
8.10	2.18	2.42	2.08	1.92	-14.19	-20.82
5.72	2.10	2.58	2.62	2.24	1.41	-13.22
13.24	1.32	3.84	5.11	4.70	33.01	22.45
8.08	1.45	3.91	6.14	4.65	56.94	18.82
7.10	1.51	5.13	7.27	5.99	41.65	16.67
2.77	1.65	4.36	8.81	5.62	101.99	28.85
4.47	1.53	4.69	5.76	5.81	22.92	23.84
6.56	2.19	2.78	2.50	2.26	-9.95	-18.61
6.47	1.92	2.15	3.03	2.00	40.96	-7.00
6.90	1.55	1.35	2.50	1.54	85.08	13.94
6.83	1.34	1.25	2.69	1.65	115.12	31.74
8.61	2.50	1.05	0.76	0.72	-27.40	-31.55
7.96	1.76	1.83	1.81	1.80	-0.83	-1.64

TABLE 9 - BASE DATA SUPPLIED BY OTHER RESEARCHERS

RESEARCHER	RIVER	LOCATION	BEND No.	RADIUS OF CURVATURE (m)	MEANDER WAVELENGTH (m)	WIDTH (m)	MEAN DEPTH (m)	VELOCITY (m/s)	SLOPE ($\times 10^{-3}$)	FRICTION FACTOR	DISCHARGE (cumecs)	SINUOSITY
Hossein	Lower Ganges	Bangladesh	1	8250	27500	5700	4.73	1.09	0.0518	0.016	29500	1.23
Hossein	Lower Ganges	Bangladesh	2	13250	37500	8240	2.65	1.35	0.0518	0.006	29500	1.2
Hossein	Lower Ganges	Bangladesh	3	19000	36000	7160	2.63	1.57	0.0518	0.004	29500	1.2
Hossein	Lower Ganges	Bangladesh	4	12500	37500	8490	2.56	1.36	0.0518	0.006	29500	1.16
Hossein	Lower Ganges	Bangladesh	1A	9750	32750	6735	4.5	0.97	0.0518	0.019	29500	1.37
Hossein	Lower Ganges	Bangladesh	2A	12000	41250	7500	2.62	1.50	0.0518	0.005	29500	1.3
Hossein	Lower Ganges	Bangladesh	3A	21250	47600	7550	3.8	1.03	0.0518	0.015	29500	1.17
Hossein	Lower Ganges	Bangladesh	4A	15500	46500	7420	3.95	1.01	0.0518	0.016	29500	1.14
Hossein	Lower Ganges	Bangladesh	1B	9750	30750	6950	4.2	1.01	0.0518	0.017	29500	1.59
Hossein	Lower Ganges	Bangladesh	2B	7500	32000	6750	3.75	1.17	0.0518	0.011	29500	1.38
Hossein	Lower Ganges	Bangladesh	3B	20000	40250	7110	3.55	1.17	0.0518	0.011	29500	1.18
Hossein	Lower Ganges	Bangladesh	4B	18500	36750	7250	4.42	0.92	0.0518	0.021	29500	1.11
Das	Malaijore	India	P	443	1920	132	2.6	2.86	1.32	0.033	982	1.44
Das	Malaijore	India	Q	181	1060	90	2.4	4.55	1.32	0.012	982	1.26
Das	Malaijore	India	R	442	1220	117	4.3	1.95	0.64	0.057	982	1.26
Das	Malaijore	India	S	242	990	129	3.2	2.38	0.66	0.029	982	1.28
Das	Malaijore	India	T	322	1220	138	3.4	2.09	1.08	0.066	982	1.31
Das	Kharasuna	India	A	692	1936	299	3.66	1.79	0.23	0.021	1964	1.43
Das	Kharasuna	India	B	1802	3613	415	3.66	1.29	0.23	0.040	1964	1.3
Das	Brahmani	India	B	644	1806	451.1	3	0.87	0.27	0.084	1177	1.57
Das	Brahmani	India	C	676	1871	219.5	3	1.79	0.27	0.020	1177	1.83
Das	Brahmani	India	D	805	2065	201.2	3	1.95	0.27	0.017	1177	1.69
Das	Haradajore	India	A	121	900	90	3.6	1.35	1.35	0.210	437	1.6
Das	Haradajore	India	B	201	1440	135	3.2	1.01	1.26	0.310	437	1.36
Das	Haradajore	India	C	684	900	115	2.6	1.46	0.7	0.067	437	1.11
Chang	San Lorenzo	California	1	152.4	646.6	67.06	2.33	2.17	2.059	0.080	339.8	1.421
Chang	San Lorenzo	California	2	171.1	554.2	67.06	2.87	1.77	2.059	0.140	339.8	1.754

TABLE 10 - CALCULATED DATA FOR BASE DATA SUPPLIED OTHER RESEARCHERS

Rc/w	dmax/dbar	MEASURED dmax (m)	BRIDGES dmax (m)	ODGAARD'S dmax (m)	EMPIRICAL dmax (m)	BRIDGE'S ERROR (%)	ODGAARD'S ERROR (%)	EMPIRICAL ERROR (%)
1.45	3.44	16.25	19.3	4.73		-70.89	18.77	
1.61	5.25	13.9	11.16	2.65		-80.93	-19.71	
2.65	5.02	13.2	10.41	2.63	5.66	-80.08	-21.14	-57.15
1.47	8.30	21.25	10.36	2.56		-87.95	-51.25	
1.45	2.93	13.17	21.03	4.50		-65.83	59.68	
1.60	6.56	17.2	10.85	2.62		-84.77	-36.92	
2.81	4.91	18.65	12.41	3.80	8.01	-79.62	-33.46	-57.03
2.09	3.99	15.75	12.25	3.95	9.99	-74.92	-22.22	-36.56
1.40	2.82	11.85	22.53	4.20		-64.56	90.13	
1.11	5.24	19.65	17.88	3.75		-80.92	-9.01	
2.81	4.00	14.2	12.65	3.55	7.49	-75.00	-10.92	-47.27
2.55	3.13	13.85	14.7	4.42	9.65	-68.09	6.14	-30.33
3.36	1.23	3.2	6.275	2.70	5.23	-15.57	96.09	63.49
2.01	1.42	3.4	5.8459	2.68	7.02	-21.28	71.94	106.47
3.78	1.77	7.6	11.3169	4.56	8.43	-40.02	48.91	10.93
1.88	1.75	5.6	10.5075	3.45		-38.35	87.63	
2.33	1.53	5.2	10.4321	3.62	7.75	-30.41	100.62	48.99
2.31	2.40	8.78	14.874	3.70	8.38	-57.83	69.41	-4.55
4.34	1.80	6.58	11.2506	3.67	6.98	-44.20	70.98	6.15
1.43	2.15	6.46	17.1651	2.95		-54.41	165.71	
3.08	2.60	7.8	11.6693	3.04	6.17	-60.99	49.61	-20.95
4.00	2.44	7.32	9.991	3.04	5.81	-58.48	36.49	-20.57
1.34	1.44	5.2	11.8587	4.55		-12.50	128.05	
1.49	1.75	5.6	8.9995	3.48		-37.86	60.71	
5.95	1.69	4.4	6.5027	2.68	4.70	-39.15	47.79	6.90
2.27	1.92	4.47	7.2675	2.90	5.40	-35.08	62.58	20.77
2.55	1.51	4.33	11.2091	3.58	6.27	-17.22	158.87	44.70

TABLE 11 - BASE DATA FOR SELECTED FLUME CHANNELS

RESEARCHER	STUDY	LOCATION	BEND No	RADIUS OF CURVATURE (m)	MEANDER WAVELENGTH (m)	WIDTH (m)	MEAN DEPTH (m)	MEAN VELOCITY (m/s)	SLOPE (X 10 ⁻³)	FRICTION FACTOR	DISCHARGE (cumecs)	SINUOSITY
Onishi et al.	III IR	USA	1	8.53	26.8	2.34	0.13	0.37	2.43	0.182	0.11	1.11
Onishi et al.	III IR	USA	2	9.12	28.6	1.17	0.13	0.37	1.80	0.138	0.06	1.11
Struiksmä	T3	Holland	A	1.64	12.31	1.09	0.062	0.36	2.58	0.095	0.02	1.365
Struiksmä	T3	Holland	B	7.76	19.44	1.09	0.062	0.36	2.58	0.095	0.02	1.167
Struiksmä	T3	Holland	C	2.72	13.61	1.09	0.062	0.36	2.58	0.095	0.02	1.189
Struiksmä	T4	Holland	A	1.76	12.31	1.09	0.082	0.42	2.61	0.095	0.04	1.365
Struiksmä	T4	Holland	B	7.76	19.44	1.09	0.082	0.42	2.61	0.095	0.04	1.167
Struiksmä	T4	Holland	C	2.72	13.61	1.09	0.082	0.42	2.61	0.095	0.04	1.189

TABLE 12 - CALCULATED DATA FOR SELECTED FLUME CHANNELS

Rc/w	MEASURED		BRIDGE'S		EMPIRICAL		BRIDGE'S	EMPIRICAL
	dmax/dbar	dmax (m)	dmax (m)	dmax (m)	dmax (m)	ERROR (%)		
3.65	2.59	0.34	0.26	0.26	0.26	-22.97	-23.03	
7.79	1.08	0.14	0.19	0.23	0.23	32.55	61.92	
1.50	2.81	0.17	0.17	0.10	0.10	-2.82	-40.41	
7.12	1.81	0.11	0.11	0.10	0.10	-3.04	-7.42	
2.50	2.60	0.16	0.13	0.10	0.10	-16.34	-35.60	
1.61	2.40	0.20	0.22	0.14	0.14	13.15	-30.39	
7.12	1.71	0.14	0.14	0.14	0.14	2.57	-2.05	
2.50	2.30	0.19	0.18	0.14	0.14	-5.71	-27.44	

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